Exercises on straight lines

* Any two pieces of info. sufficient to find equation of a straight line.

* Every straight line can be written as: $y = m x + b$.

* Every linear equation in 2 variables is a straight line.

(I) How to find the equation of line using slope & any 1 point on it:

Exercise 60 (pg. 183): Find line that passes through (-2, -5) with slope of 3/4.

Strategy: We know m = 3/4, so we get $y = \frac{3}{4}x + b$.

To find b, plug in coordinates of point that it passes through & solve.

(II) How to find the equation of line using any 2 points on it:

Exercise 70 (pg. 183): Find the line that passes through (-1, 4) and (6, 4).

Strategy: Find slope (of segment) that connects the 2 points, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

To find b, plug in coordinates of any one point.

You can use the 2nd point to verify your answer.

(III) How to find the equation of line using its x-intercept & y-intercept:

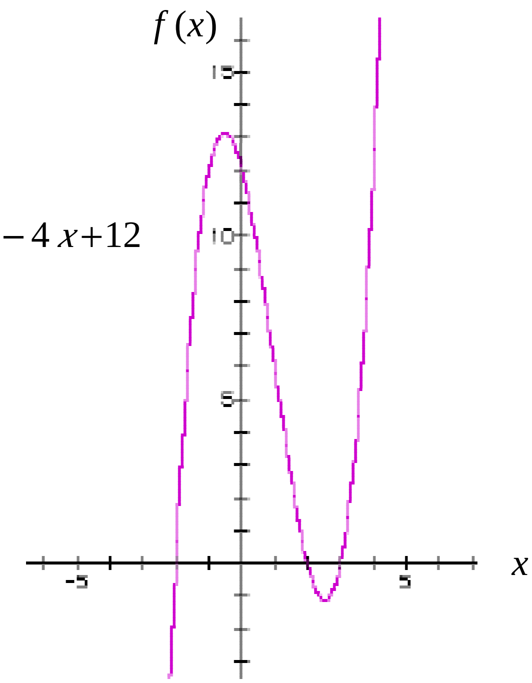
Exercise 84 (pg. 183): Find the line with x-intercept = 2/3, y-intercept = -2.

Strategy: Find slope (of segment) that connects the 2 intercepts, $m = \dfrac{y_2 - 0}{0 - x_1}$.

We are given the y-intercept, so we can directly plug into $y = m\,x + b$.

Function graphs: Key things to observe

$f(x) = x^3 - 3x^2 - 4x + 12$



(1) Zeros: x-values where graph crosses the x-axis.

(2) Increasing/Decreasing intervals: Moving left to right,

any x-interval where graph is rising $\Rightarrow$ increasing

x-interval where graph is falling $\Rightarrow$ decreasing

(3) Relative extrema (minimum/maximum):

any x-value where graph reaches a peak $\Rightarrow$ maximum

x-value where graph reaches valley $\Rightarrow$ minimum

A function always switches direction (e.g., incr. to decr.) at such points.

(4) Even & odd functions:

graph is symmetric with respect to y-axis $\Rightarrow$ even $[\, f(x) = f(-x) \,]$

symmetric with respect to origin $\Rightarrow$ odd $[\, f(x) = -f(-x) \,]$

Q: What is symmetry with respect to x-axis called?

Some function "Role Models"

Objective: To learn to recognize the graph & algebraic form of a handful of special functions that shall serve as "role models" for all future functions!

We consider the following specific functions:

(1) $f(x) = 1$ [$= x^0$]

(2) $f(x) = x$ [$= x^1$]

(3) $f(x) = x^2$

(4) $f(x) = x^3$

(5) $f(x) = \sqrt{x}$ [$= x^{1/2}$]

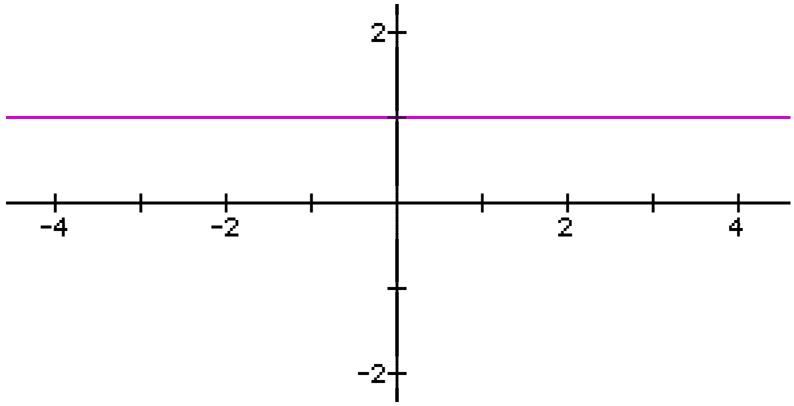
(6) $f(x) = \frac{1}{x}$ [$= x^{-1}$]

(7) $f(x) = |x|$

For each case, try to remember the following specific attributes:

* Graph shape
* Domain & range (e.g., is it always positive, always negative, or both)
* Where is the graph increasing & where is it decreasing.
* What kind of symmetry, if any.

(1) f (x) = 1: Constant case:

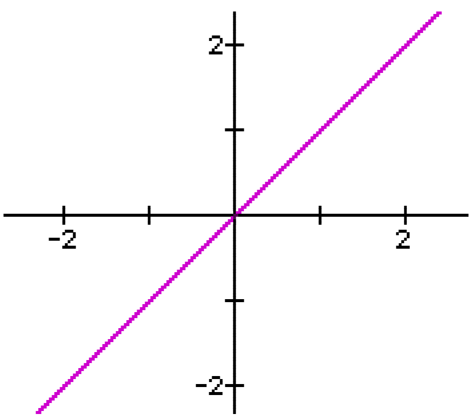


* Graph is Horizontal straight line. y=1 for all values of x.

* Domain is all reals. Range is y=1.

* Symmetry about y-axis (i.e., even function).

(2) f (x) = x: Linear case:

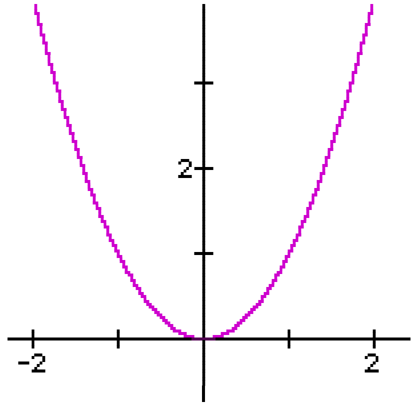


* Graph is straight line with positive slope=1.
* Domain & range include all reals.

* Increases on $(-\infty, \infty)$

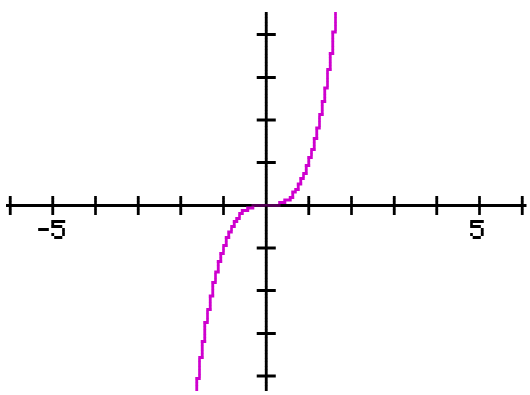
* Symmetry about origin (i.e., odd function)

(3) $f(x) = x^2$: Quadratic case:



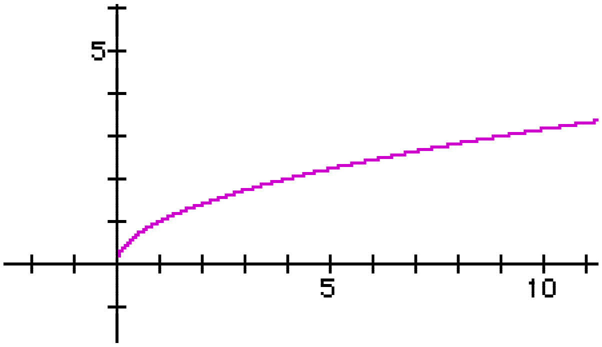
* Graph is parabola - opens up - nose at origin.
* Domain: all reals; Range: only positive reals.
* Decreases $(-\infty, 0)$; increases $(0, \infty)$
* Symmetry about y-axis (i.e., even function).

(4) $f(x) = x^3$: Cubic case:



* Graph shape is like a "stretched out" N.
* Domain & Range: all reals.
* Increases on $(-\infty, \infty)$
* Symmetry about origin (i.e., odd function).

(5) f (x) = √ x: Square root case:



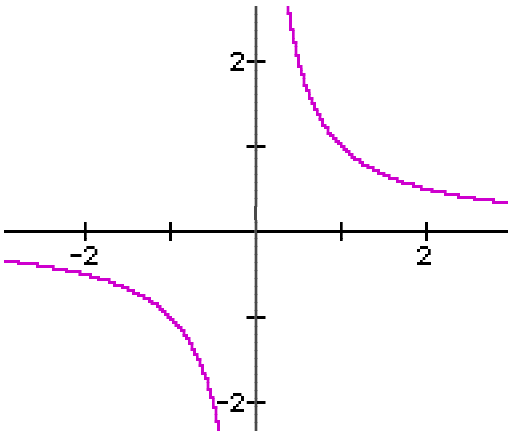
* Graph looks like a slowly ascending airplane! Starts from origin.

* Domain & Range: only positive reals.

* Increases on (0, ∞)

* Symmetry: None.

(6) f (x) = 1/ x: Reciprocal case:



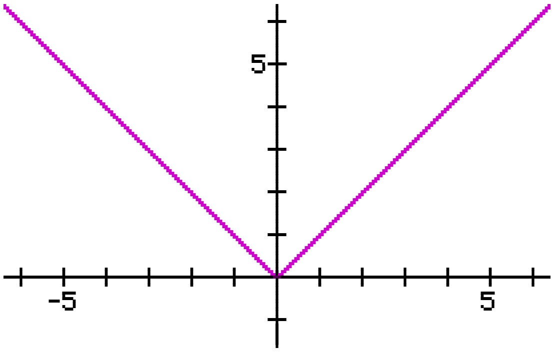
* Graph like "bowls" in 1st and 3rd quadrants.

* Domain & Range: all reals except 0.

* Decreases: (-∞, 0), decreases: (0, ∞)

* Symmetry: about origin (odd function)

(7) f (x) = |x|: Absolute value case:



* Graph like a V-shape - base at origin.

* Domain: all reals. Range: only + reals.

* Decreases: $(-\infty, 0)$ Increases: $(0, \infty)$

* Symmetry: about y-axis (even function)

How to move and/or stretch functions

Objective: To learn the effect of certain common algebraic operations on the graphs of functions.

We look at 3 types of effects on functions:

(1) How to move the graph up/down or left/right like a rigid body

(2) How to create mirror reflections of graphs

(3) How to shrink or stretch the graph horizontally or vertically

[View powerpoints from textbook website for good illustration of concept.]

Summary of key results

Translation

(1) If you add (or subtract) a constant c to $f(x)$, you move it up (or down).

e.g., x^3-2 is just x^3 shifted down by 2 units

$|x-2|+3$ is just $|x-2|$ shifted up by 3 units

(2) If you replace every x by $x+c$ or $x-c$ you shift the graph left or right.

e.g., $(x-2)^3$ is just x^3 shifted right by 2 units

$|x+2|$ is just $|x|$ shifted left by 2 units

Reflection

(1) If you flip the sign of every term in $f(x)$, you get its reflection about x-axis.

e.g., x^3-2 is just the reflection of $-x^3+2$ about the x-axis

$-|x+2|$ is just the reflection of $|x+2|$ about the x-axis

(2) If you replace every x by $-x$ you get the graph's reflection about the y-axis.

e.g., x^3-2 is just the reflection of $-x^3-2$ about the y-axis

$|x+2|$ is just the reflection of $|-x+2|$ about the y-axis

Stretching & shrinking

(1) If you multiply $f(x)$ by a constant c it stretches or compresses vertically.

e.g., $4x^3$ is just the graph of x^3 stretched vertically (in y-direction)

$-|x+2|$ is just the reflection of $|x+2|$ about the x-axis

(2) If you replace every x by $c\,x$ it stretches or compresses horizontally.

e.g., $|4x+2|$ is just $|x+2|$ compressed horizontally

Inverses of functions

Objective: To understand what is the inverse of a function & to learn how to find it.

Summary of key ideas

* An inverse is very much like working with a function backwards -- i.e., if $y = f(x)$, we look for a relationship that takes y as input and gives back x.

* Many functions have inverse functions, but not all do.

* The inverse relationship works both ways when a function does have an inverse. i.e., if $g(x)$ is the inverse of $f(x)$, then $f(x)$ is the inverse of $g(x)$.

* The domain & range get reversed -- if $g(x)$ is the inverse of $f(x)$, then the domain of g is the range of f and vice versa.

* Composition result: Since the inverse function essentially "undoes" the effect of the orginal function, when we use them back-to-back x remains unchanged. That is: $f(g(x)) = x$ and $g(f(x)) = x$.

* Existence of inverse: An inverse exists for any function only provided it is one-to-one. This means the function must not only be single-valued, but also may not have multiple x-values that give the same y-value.

e.g., $f(x) = x^2$ does not have an inverse since x and $-x$ give the same y-value.

* Notation: The inverse of any function $f(x)$ is written as $f^{-1}(x)$.