Algebraic strategies for 1-variable equations

We first consider linear and quadratic equation types.

(1) Linear

* This is the easiest case.
* Any linear equation can be rearranged to look like: $a x + b = 0$ (where a and b are some numbers).

  E.g., $10 = 4 - 3x$ is the same as $3x + 6 = 0$ OR $0 = -6 - 3x$
* Can readily solve for x: $x = -b / a$

Quadratic

* Any quadratic can be rearranged to look like: $a x^2 + b x + c = 0$ (where a, b and c are numbers).
* Some quadratics are much easier to solve without re-writing in this form (e.g., by factoring, or other short-cuts).
* However, every quadratic can be solved if we write it in the above form, by using the famous "quadratic formula" which is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

* There are very few things in math I recommend memorizing -- the quadratic formula is one of them!

Solving other 1-variable equations

Other equation types:

(1) Polynomials. [Powers of x combined linearly: $2x^6 - x^5 - 3x^4 + 12x = 0$]

(2) Rationals. [Ratio of 2 polynomials: $\dfrac{x}{2x^6 - x^5 - 3x^4 + 12x} = 0$]

(3) Equations involving square roots or other roots (i.e., radicals).

(4) Equations involving absolute values.

Background notes:

* Only linears and quadratics have guaranteed solution methods.
* For other equations, we try to isolate the unknown using one or more of the standard algebraic manipulations.
* Graphing software and/or calculator is extremely useful here.

Polynomials: Key strategies to try.

(1) Factoring (sometimes, repeatedly in 2 or 3 stages)

(2) Making the equation "look like a quadratic."

Rationals:

* By definition, these are ratios, which means fractions that contain variables!

* Strategies involve the standard tricks that we play with fractions, such as finding common denominators, cross-multiplying &/or multiplying through by a common factor, etc.

Very important algebraic fact:

An equation in fraction form such as

$$\frac{a}{b} = 0 \qquad (a \text{ and } b \text{ being any algebraic expressions})$$

necessarily requires that $a=0$ for the equation to be true.

Q: Can you see why this is so? Hint: Look at it as a product: $a * \frac{1}{b} = 0$

* This powerful result is frequently used when solving rational equations.

Square roots & other radicals: Key strategies to try.

(1) Isolate the square-root term on 1 side & square both sides.

(If it is any other type of root, follow very similar strategy.)

(2) In some cases rationalization may help.

What is rationalization?

* It is very much like the “complex conjugate” idea.

* Makes use of the very versatile identity: $(x+y)(x-y)=x^2-y^2$

* Learn to recognize this in various other incarnations: (some examples)

$$(1+x)(1-x)=1-x^2$$

$$(x^2+y^2)(x^2-y^2)=x^4-y^4$$

$$(1+\sqrt{x})(1-\sqrt{x})=1-x$$

$$(\frac{1}{\sqrt{x}}+1)(\frac{1}{\sqrt{x}}-1)=\frac{1}{x}-1$$

$$(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})=x-y$$

* These are conjugate factors.

* Rationalization involves the clever use of conjugates to simplify algebraic expressions containing square roots in fractions.

Absolute values in equations:

The **main point** to remember here is that absolute value equations are best viewed as 2 separate equations.

How?

Suppose: $|8-3x|=4$

This really means two things: $8-3x=4$ and $8-3x=-4$

Therefore, we must solve each equation independently.