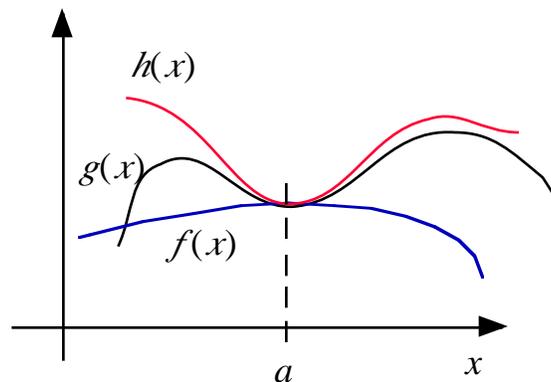


## A general note on theorems

- \* Recall, a key objective of mathematics is to discover patterns and relationships.
- \* Theorems are, essentially, a general statement of a relationship that can be logically proven starting from some minimum set of accepted facts (axioms).
- \* Most theorems look like the following statement:  
"If **A**, then **B**."  
"If *hypothesis*, then *conclusion*."  
E.g., "If two odd numbers are added, then the result is an even number."
- \* The hypothesis and the conclusion are both key components of a theorem. It doesn't make sense to omit (or misstate) either part.
- \* Some theorems have a long and detailed hypothesis, with a very short conclusion. The Squeeze Theorem is a perfect example of this!

### The Squeeze Theorem



#### Hypothesis:

- \* Suppose we want to find  $\lim_{x \rightarrow a} g(x)$  for some function  $g(x)$ .
- \* Suppose we can find two other functions  $f(x)$  and  $h(x)$  that completely enclose  $g(x)$  from above and below when  $x$  is in the neighborhood of  $a$ .  
In mathematical terms:  $f(x) \leq g(x) \leq h(x)$  in some open interval containing  $a$  except, possibly, at  $a$  itself.
- \* Lastly, suppose  $\lim_{x \rightarrow a} f(x)$  and  $\lim_{x \rightarrow a} h(x)$  have the same numerical value, say  $L$ .

#### Conclusion:

- \* Then  $\lim_{x \rightarrow a} g(x) = L$ . In other words,  $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = \lim_{x \rightarrow a} h(x)$ .