Test 3 practice worksheet

1. State the Mean Value Theorem in mathematically precise language, followed by a brief explanation of what it means in everyday language. Include a graph, with labels, to illustrate your claims.

2. State the mathematical definition of the definite integral of a function f.

3. Give a mathematically precise statement of both parts of the Fundamental Theorem of Calculus. Explain why the theorem is considered significant, or "fundamental."

4. Compute using the evaluation theorem: $\displaystyle\int_{\pi}^{2\pi} \left(\frac{1 - 6x + 3x\cos x}{3x} \right) dx$

5. Indicate true or false and justify your answer. If you say "True" give reason(s). If you say "False" give an example in which the statement doesn't hold – you may give your example in the form of a clearly-labeled graph.

   (a) If $f(x)$ has a local minimum value at $x = c$, then it is necessary that $f'(c) = 0$ or $f'(c) =$ undefined.

   (b) If a function is continuous on a closed interval, then it is guaranteed to attain local minimum and maximum values on that interval.

   (c) If $f(x) \leq g(x)$ on the interval $[a, b]$, then it is necessary that $f'(x) \leq g'(x)$ on that interval.

   (d) If $f(x) \leq g(x)$ on the interval $[a, b]$, then it is necessary that $\displaystyle\int_a^b f(x)dx \leq \int_a^b g(x)dx$ on that interval.

6. A particle moves back and forth along a straight line. Its velocity (in meters per second) at time t (seconds) is given by the function $v(t) = 2t^2 + 2t - 12$. Find the total distance traveled by the particle during the interval $0 \leq t \leq 4$.

7. Evaluate the following limits: $\displaystyle\lim_{x \to -\infty} \frac{1 - e^x}{1 + e^x}$ and $\displaystyle\lim_{x \to \infty} \frac{1 - e^x}{1 + e^x}$.

   Show solution steps and reasons.

8. Find the absolute minimum and maximum values of the function $y = \dfrac{3 - x^2}{e^x}$ on the interval $[-2, 0]$.

9. Find the linear approximation of $g(t) = \sqrt[3]{t^2 - 1}$ at $t = 3$ and use it to approximate $\sqrt[3]{2.9}$ and $\sqrt[3]{3.05}$.

10. Find dy/dx for each the following:

    (a) $y = \dfrac{1}{x \, \ln x}$

    (b) $y = (1 + 2x)^{1/x}$

    (c) $y = \displaystyle\int_x^2 \sqrt{3 + 5t^2} \, dt$

    (d) $y = \displaystyle\int_x^{e^x} \sqrt{3 + 5t^2} \, dt$

11. Let V be the volume of a cylinder having height h and radius r, where both h and r vary with time. When the height is 6 in. and is increasing at 0.2 in./sec., the radius is 4 in. and is decreasing at 0.1 in./sec. Find the rate at which the volume is changing, and determine whether it is increasing or decreasing at that instant.

12. A piece of wire 20 ft long is to be cut into two pieces. One piece is to be bent into an equilateral triangle, and the other into a circle. Determine the length of each piece so that the sum of the areas is maximized.

13. Suppose $v(t)$ is the velocity (in meters per second) of an object moving back and forth along a straight line. What physical meaning is associated with each of the following

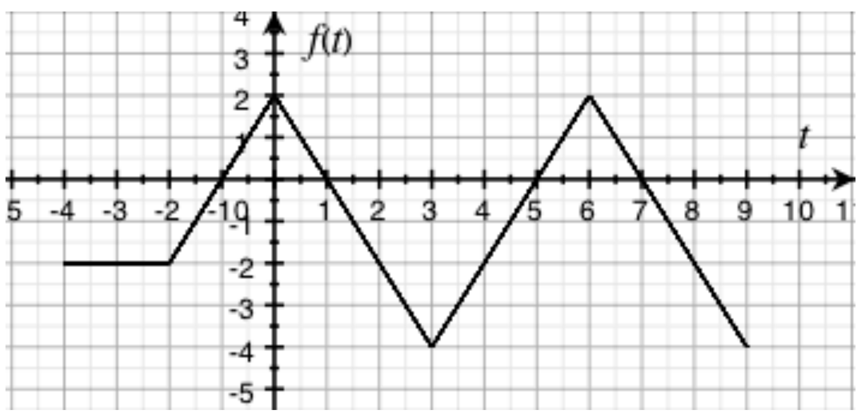
    (a) $\int_1^5 v(t)\, dt$

    (b) $\int_1^5 |v(t)|\, dt$

    (c) $\frac{1}{5-1}\int_1^5 v(t)\, dt$

    (d) $v'(t)$

14. Let $g(x) = \int_{-1}^{x} f(t)dt$, where the graph of f is shown below.



    (a) Determine the sign of $g(-2)$, $g(1)$ and $g(5)$ [positive or negative]. Explain your reasoning.

    (b) Find the value of $g(3)$ and $g'(3)$. Give reasons.

    (c) On $-4 < x < 9$ on what interval(s) is $g(x)$ increasing, and on what interval(s) is it decreasing?

    (d) On $-4 \leq x \leq 9$ at what x-values does $g(x)$ have absolute minimum and maximum values? Why?

15. Find the most general antiderivative of the following functions:

    (a) $y = \dfrac{x^2 - \sqrt{x}\, e^x}{\sqrt{x}}$

    (b) $g(x) = 2\cos x + \dfrac{1}{4x}$

    (c) $f(x) = \left(2x - \dfrac{1}{\sqrt{x}}\right)\left(1 + \dfrac{1}{\sqrt{x}}\right)$

    (d) $h(x) = \dfrac{4}{1+x^2} - 3x^2$