Quiz 8 - 4/20/2022

Find dy/dx for: $y = (\sin x)^x$

Solution

Take "ln" on both sides in order to move variables out of the exponents:

$$\ln(y) = \ln(\sin x)^x \quad \Rightarrow \quad \ln(y) = x \cdot \ln(\sin x)$$

Next, differentiating both sides with respect to x, we have

$$\frac{1}{y}\frac{dy}{dx} = \frac{dx}{dx} \cdot \ln(\sin x) + x \cdot \frac{d}{dx}\ln(\sin x)$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = 1 \cdot \ln(\sin x) + x \cdot \left(\frac{1}{\sin x}\right) \cos x$$

Solve for dy/dx:

$$\frac{dy}{dx} = y\left[\ln(\sin x) + x \cdot \frac{\cos x}{\sin x}\right]$$

$$\therefore \boxed{\frac{dy}{dx} = (\sin x)^x \ [\ln(\sin x) + x \cot x]}$$

Grading: Total points possible = 6.

1pt = apply ln on both sides and get $\ln y = x \cdot \ln(\sin x)$
1pt = correct implicit derivative of $\ln y$
3pt = correct derivatives of $x \cdot \ln(\sin x)$
1pt = multiply by y and replace it with $(\sin x)^x$