

Quiz 6 - 3/29/2022

(I) Differentiate with respect to x and simplify: $y = \frac{5x^2\sqrt{x} - 3x + 1}{\sqrt{x}}$.

["Differentiate" means "find derivative."]

(II) Suppose $f(1) = 2$ and $f'(1) = -3$. Find the derivative of $x^2f(x)$ at $x = 1$.

Solution

(I) The simplest strategy for this is to do some preliminary algebra and rewrite it in a form suitable for applying the power rule:

$$y = \frac{5x^2\sqrt{x} - 3x + 1}{\sqrt{x}} = (5x^2 \cdot x^{1/2} - 3x + 1) \cdot x^{-1/2} = 5x^2 - 3x^{1/2} + x^{-1/2}$$

$$\text{Therefore, } y' = 10x - 3 \cdot \frac{1}{2}x^{-1/2} + 1 \cdot \left(-\frac{1}{2}x^{-3/2}\right) =$$

$$\boxed{10x - \frac{3}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}} \quad \text{OR} \quad \frac{20x^2\sqrt{x} - 3x - 1}{2x\sqrt{x}}} \quad (\text{answer})$$

Method 2: Via quotient rule:

$$y = \frac{5x^2\sqrt{x} - 3x + 1}{\sqrt{x}} \Rightarrow y' = \frac{(5x^{5/2} - 3x + 1)' \sqrt{x} - (5x^{5/2} - 3x + 1)(\sqrt{x})'}{x}$$

$$= \frac{[(25/2)x^{3/2} - 3] \sqrt{x} - (5x^{5/2} - 3x + 1)(\frac{1}{2\sqrt{x}})}{x}$$

$$= \frac{(25x^{5/2} - 6x) - (5x^{5/2} - 3x + 1)}{2x\sqrt{x}} = \frac{20x^2\sqrt{x} - 3x - 1}{2x\sqrt{x}}$$

$$\boxed{= 10x - \frac{3}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}}} \quad (\text{same answer as before!})$$

(II) $[x^2f(x)]' = x^2f'(x) + (x^2)'f(x) = x^2f'(x) + 2xf(x)$

At $x = 1$ this becomes: $1^2 \cdot f'(1) + 2 \cdot 1 \cdot f(1)$

$$= 1 \cdot (-3) + 2 \cdot 2 \quad \boxed{= 1}$$

Grading: Total points possible = 6.

0.5 pt - Any reasonable attempt.

3.5 pt for (I): 1.5 pt = correctly rewrite as powers of x .

1.5 pt = apply power rule correctly.

0.5 pt = simplify.

For quotient rule method:

1 pt = correctly plug into QR formula.

1.5 pt = correctly find derivatives in the numerator.

1 pt = simplify to a correct form of the answer.

2 pt for (II): 1 pt = find correct 2 terms in derivative of $x^2f(x)$.

1 pt = plugin given numbers and correctly evaluate result.