Worksheet 8

1. Differentiate each of the following functions and simplify:

   (a) $y = \tan^3 x$

   (b) $z = \tan(x^3)$

   (c) $f(x) = \sqrt{x - x^2}$

   (d) $g(x) = \sqrt{x - x^2}\, e^{3x}$

   (e) $h(x) = \sqrt{x - x^2}\, e^{\sin(3x)}$

   (f) $v(t) = \sin(t\, e^t)$

   (g) $w(t) = \dfrac{\sin(t\, e^t)}{t^3 + t}$

   (h) $w(t) = (t^3 + t)^3 \sin(t\, e^t)$

   (i) $y = \left(\dfrac{x+1}{x-3}\right)^4$

   (j) $f(x) = \sin^3(2x) + \cos^3(x)$

   (k) $g(x) = \sin^3(2x) \cdot \cos^3(x)$

   (l) $h(x) = \sqrt{\sin^3(2x) + \cos^3(x)}$

2. Suppose $h(x) = f(g(x))$ and $r(x) = g(f(x))$, and we are given the information in the following table

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-4	0	-5	0	5
-2	2	-2	4	3
0	4	2	6	1
2	-6	1	6	-1
4	-4	3	4	-3
6	0	5	0	-5

   (a) Find $h'(-2)$ and $h'(2)$.

   (b) Find $r'(-2)$ and $r'(4)$.

   (c) Suppose $s(x) = f(g(f(x)))$. Find $s'(0)$.

3. Find dy/dx for each of the following:

   (a) $y = \tan^3(\sin x)$

   (b) $x = t^3 - 3t^2 + 1, \quad y = \dfrac{1}{t\sqrt{t}}$

   (c) $x = 2\sin(3t), \quad y = \cos(3t)$

   (d) $x = 2\sin^2(3t), \quad y = \cos^3(3t)$

   (e) $x = r\sin(\theta - \sin\theta), \quad y = r(1 - \cos(\theta))$ with r being a constant.

   (f) $y = A\cos(\omega x + \delta)$
   Here A, ω, δ are constants.

4. Find solutions to each of the following, as instructed.

a) Find an equation of the tangent line to the curve $x = 2\sin(t) + 5, y = 4 - 5\cos(t)$ at $t = \frac{5\pi}{4}$.

b) Find the (x, y) coordinates of the point(s) where the curve $x = 2\sin(t) + 5$, $y = 4 - 5\cos(t)$ has horizontal tangent lines.

c) Find an equation of the tangent line to the curve $x = \cos(\theta) - \sin(2\theta)$, $y = \sin(\theta) + \cos(2\theta)$ at $\theta = 2$.

d) Show that the curve $x = \sin t$, $y = \sin(t + \sin t)$ has two tangent lines at the origin, and find their equations.