

Worksheet 7

1. Solve the following trigonometric equations:

(a) $\cos x - 1 = 0$

(b) $\cos^2 x - 1 = 0$

(c) $2 \cos^2 x - 1 = 0$

(d) $\cos^2 x - 2 \cos x + 1 = 0$

(e) $\cos x = \sin x$

(f) $\cos^3 x = \cos x$

(g) $\sin x - 2 \sin x \cos x = 0$

(h) $\cos^2 x = 1 - \sin x$

2. Differentiate each of the following functions and simplify:

(i) $f(x) = 5x\sqrt{x} - 2x^{5/2} + \sqrt{x}$

(ii) $g(x) = \frac{5x\sqrt{x} - 2x^{5/2} + \sqrt{x}}{x}$ (Hint: Avoid using the quotient rule!)

(iii) $h(x) = \frac{5x\sqrt{x} - 2x^{5/2} + \sqrt{x}}{x\sqrt{x}}$ (Hint: See previous hint!)

(iv) $f(t) = \frac{t^2 + 1}{t^2 - 1}$

(v) $g(t) = \frac{e^t}{3 + t^2}$

(vi) $h(t) = \frac{e^t}{3 + t e^t}$

(vii) $r(t) = (t^3 - 5t)e^t$

(viii) $f(u) = (u^4 - 4u^3 + 8u - 3)(2u^3 - 3u + 4)$

(ix) $g(u) = \frac{(u^4 - 3)(2u^3 + 4)}{\sqrt{u}}$ (Hint: Avoid the QR!)

(x) $f(x) = e^x + \frac{x^e + 1}{x^\pi}$

3. Let $g(x)$ be a differentiable function. Find $f'(x)$ for each of the following in terms of $g(x)$ and $g'(x)$:

(a) $f(x) = e^x g(x)$

(b) $f(x) = \frac{3x - 1}{g(x)}$

(c) $f(x) = g(x)(\sqrt{x} - 8x^2)$

(d) $f(x) = g(x) \left(\frac{\sqrt{x} - 8x^2}{x} \right)$

4. Find solutions to each of the following, as instructed.

- a) Find an equation of the tangent line to the curve $y = e^x(x + 1)$ at $x = 0$.
- b) Find the point(s) where the curve $y = \frac{x}{x^2 + 4}$ has horizontal tangent lines.
- c) Given $f(x) = x^4e^x$, find the intervals where the graph of f is concave up.
- d) Find an equation of the tangent line to the curve $y = \frac{1 - x}{1 + x}$ at $x = 3$.
- e) Find equations of both tangent lines to the curve $y = \frac{1 - x}{1 + x}$ that are parallel to the line $2x + y = 3$.
- f) Find an equation of the tangent line to the curve $y = e^x$ that passes through the origin.
- g) Find the equation of the line(s) tangent to the graph of $y = 3x^2 - 5$ and parallel to the line $7x - y = -1$.