Worksheet 12

1. Find all the critical points of each of the following functions:

   (a) $y = x^3 - 3x + 27$
   (b) $f(x) = |x^2 - 9|$
   (c) $G(t) = \sqrt[3]{25 - x^2}$
   (d) $y = 2\sin x - x$
   (e) $F(t) = t^3 - t^2 - t$
   (f) $f(x) = \dfrac{e^{-x}}{1 + x^2}$
   (g) $y = |x|\ (x^2 - 4)$
   (h) $y = x\ |x^2 - 4|$
   (i) $g(t) = 2\sin t\cos t$

2. Each of the following questions requires sketching the graph of a function f that has all the indicated properties:

   (a) f is defined on $[-2, 5]$, has no local extreme values, but does have absolute minimum and maximum values.
   (b) f is defined on $[-2, 5]$, has no absolute extremes, but does have local minimum and maximum.
   (c) f is defined on $[-2, 5]$, has two critical points, but has neither local nor absolute extremes.
   (d) f is differentiable everywhere, with $f'(1) = f'(4) = 0$, $f''(1) = -2$ and $f''(4) = 0$.

3. Find all the local and absolute extreme values of the following functions on the indicated interval using calculus techniques.

   a) $f(x) = |x^2 - 9|$ on $[-4, 5]$.
   b) $F(t) = t^3 - t^2 - t$ on $[-1, 2]$.
   c) $y = |x|\ (x^2 - 4)$ on $[-3, 3]$.
   d) $f(x) = (x^2 - 1)e^x$ on $[-4, 2]$.
   e) $f(x) = (x^2 - 1)e^x$ on $[-4, 2]$.
   f) $f(x) = (x + 3)^4(x - 2)^3$ on $[-4, 2]$.
   g) $f(x) = (x^2 - 1)^3$ on $(-\infty, \infty)$.

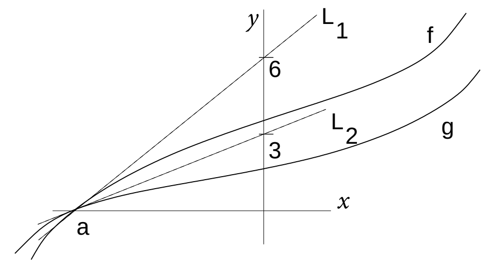
4. Find the following limits:

   a) $\displaystyle\lim_{x\to 0} \frac{\cos x - 1}{x^2}$
   b) $\displaystyle\lim_{x\to\infty} \frac{(\ln x)^2}{x}$
   c) $\displaystyle\lim_{x\to 0} \frac{e^{x^2} - 1}{x}$
   d) $\displaystyle\lim_{x\to 0^+} x\ (\ln x)^2$
   e) $\displaystyle\lim_{x\to 0}(1 + x)^{1/x}$
   f) $\displaystyle\lim_{x\to 0}(1 + x^2)^{1/x}$

5. Solve each of the following as instructed:

a) Find the intervals on which the function $y = e^{x|x-2|}$ is increasing, and on which it is decreasing.

b) Suppose f and g are two functions with linear approximations L_1 and L_2 as shown.
Find $\lim_{x \to a} \frac{f(x)}{g(x)}$.
Hint: Is it indeterminate?



c) True or false: Suppose $f''(x) < 0$ for all x near the point $x = 3$. Then the linear approximation of f at $x = 3$ will overestimate the value of $f(2.9)$.

d) Suppose f is differentiable on the interval $[1, 4]$, and suppose $0 \leq f'(x) \leq 5$ for all x in that interval. If $f(1) = 2$, what are the minimum and maximum possible values of $f(4)$?

e) State the Mean Value Theorem in mathematically precise language, followed by a brief explanation of what it means in everyday language. Include a graph, with labels, to illustrate your claims.