

Worksheet 11

- Find the linear approximation to each of the following functions at the indicated point:
 - $f(x) = e^{-3x}$ at $x = 0$
 - $f(x) = x^{2/3}$ at $x = 8$
 - $f(x) = \sin x$ at $x = \pi$
 - $f(x) = (\sin^2 x)(\cos x)$ at $x = \pi/2$
- A particle moves along a straight line with equation of motion $s = t^2 - 3t + 2$, where s is in meters and t in seconds. On the interval $0 \leq t \leq 8$:
 - Find the value(s) of t at which the particle reverses direction.
 - When is the particle speeding up, and when is it slowing down?
 - How much total distance does it travel between $t = 0$ and 8 ?
- A particle moves along a straight line with equation of motion $r = t^3 - t^2$, with r in meters and t in seconds. On the interval $0 \leq t \leq 5$:
 - Find the value(s) of t at which the particle reverses direction.
 - When is the particle speeding up, and when is it slowing down?
 - How much total distance does it travel between $t = 0$ and 5 ?
- The length of a rectangle is increasing with time at the rate of 2 feet per second, while its width is increasing at a rate of 1 foot per second. At the instant when the length is 5 feet and width is 3 feet:
 - How fast is its area changing?
 - How fast is the length of the diagonal changing?
- A plane, flying horizontally at an altitude of 1 km and speed of 500 km/hr, passes directly over a radar station. Find the rate at which the distance from the plane to the station is changing when it is 2 km away from the station.
- Find the linear approximation of $g(t) = \sqrt{t + 24}$ at $t = 1$ and use it to approximate $\sqrt{24.98}$ and $\sqrt{25.06}$.
- The profit (in \$) for a company when x units of a certain product are produced is given by $P(x) = (100 - x) \ln x$, with $x > 1$.
 - Find the marginal profit function (the derivative of the profit).
 - If the current production level is $x = 15$, is the profit increasing or decreasing?
 - If the current production level is $x = 40$, is the profit increasing or decreasing?

8. (a) Show that the rate of change in the area of a square, with respect to its side length, is half its perimeter.
- (b) Show that the rate of change in the area of a circle, with respect to its diameter, is half its circumference.
- (c) Show that the rate of change in the volume of a cube, with respect to its side length, is half its surface area.
9. Two cars are each 100 miles away from the town of Richmond, IN. One is directly to the north and heading towards town at a speed of 60 miles per hour. The 2nd car is directly to the east and heading into town at 30 miles per hour. How fast are the cars approaching each other?

10. A boat floating several feet away from a dock is being pulled in with a rope as shown. The rope is pulled in at a constant rate of 20 feet per minute, and the winch is 10 feet above the level of the boat. How fast is the boat approaching the dock when there is 100 feet of rope between the boat and the winch?

