Worksheet 14

1. Differentiate with respect to x and simplify:

   (a) $g(x) = \sqrt{1 - e^{2x}}$

   (b) $f(x) = \dfrac{5x - x^2}{\sqrt[3]{x}}$

   (c) $h(x) = (\cos x)^{\sqrt{x}}$

   (d) $r(x) = \displaystyle\int_3^x e^{t^2 - 3t} dt$

   (e) $s(x) = \displaystyle\int_{-2}^{x^2} e^{t^2 - 3t} dt$

   (f) $y = \ln \sqrt{2 - 3x}$

   (g) $y = \ln \dfrac{(2 - 3x)^3}{(3 - 4x)^5}$

2. Evaluate the following limits

   (a) $\displaystyle\lim_{x \to 1} \frac{x^2 + 4x - 5}{x - 1}$

   (b) $\displaystyle\lim_{x \to 1} \frac{\sqrt{x + 3} - 2}{x - 1}$

   (c) $\displaystyle\lim_{x \to -3} \left[\frac{1}{3 + x} - \frac{6}{9 - x^2} \right]$

   (d) $\displaystyle\lim_{x \to \infty} (x - 1)e^x$

   (e) $\displaystyle\lim_{x \to -\infty} (x - 1)e^x$

   (f) $\displaystyle\lim_{x \to \infty} \frac{1 - e^x}{1 + e^x}$

   (g) $\displaystyle\lim_{x \to -\infty} \frac{1 - e^x}{1 + e^x}$

   (h) $\displaystyle\lim_{x \to 0} \frac{\cos^2 x - 1}{x^2}$

   (i) $\displaystyle\lim_{x \to 0} x(\sin^3 x)$

3. Find the absolute minimum and maximum values of the function $g(x) = (3 - x^2)e^x$ on the interval $[0, 2]$ using calculus techniques.

4. Find the equation of the line tangent to the graph of $\ln(x) + \ln(y) = y^3 - 1$ at the point $(1, 1)$.

5. Find the equation of the line tangent to the graph of $F(x) = \displaystyle\int_1^{x^2} \sqrt[3]{2t - 1}\, dt$ at $x = 1$.

6. A rectangular display area containing 800 square feet is to be enclosed outside a shopping mall. Three sides of the enclosure are to be built using fencing that costs \$20 per foot. The 4th side is to be made using more expensive fencing that costs \$30 per foot. Find the dimensions that would minimize total cost, and find the minimum cost. [Remember: Must prove that your answer is the abs. min.]

7. Given: $\displaystyle\sum_{i=1}^{n} \left(\sqrt{1 + i\frac{8}{n}} \right) \frac{8}{n}$

   (a) Compute the sum for $n = 4$.

   (b) This represents the right Riemann sum of a function $h(x)$ over an interval. Find h and the interval.

8. Evaluate the following integral: $\displaystyle\int_0^1 \frac{5x - x^2}{\sqrt[3]{x}}\, dx$

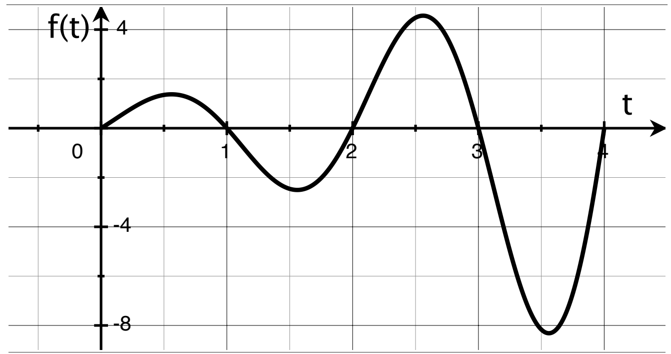
9. Let $F(x) = \int_0^x f(t)dt$ on the interval $[0, 4]$, where the graph of f is shown below.

(a) Is $F(2)$ positive or negative? Reason?

(b) On what intervals is $F(x)$ increasing? Decreasing?

(c) At what values of x does F have local minimum or maximum values? Reason?

(d) On what intervals is F concave up/down? Reason?



(e) At what values of x does F have absolute minimum or maximum values? Reason? [Hint: Do you know the critical points, end points? Can you quantify F values at those points?]

10. (a) Give a mathematically precise definition of the definite integral $\int_a^b f(x)dx$.

(b) Give a mathematically precise statement of both parts of the Fundamental Theorem of Calculus.

One more like (9) for practice:

11. Let $g(x) = \int_{-2}^x f(t)dt$, where the graph of f is shown below.

(a) Evaluate $g(7)$. Show reasoning.

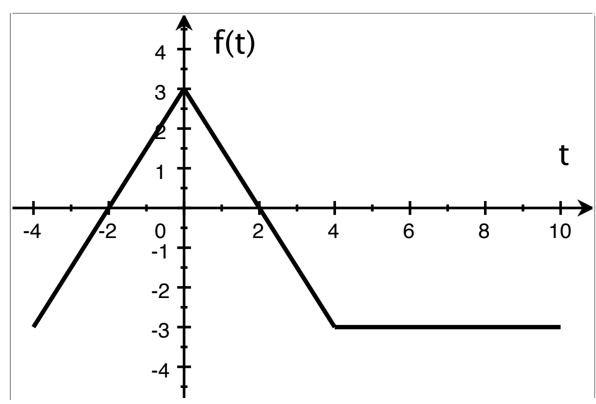
(b) Evaluate $g(0)$. Show reasoning.

(c) Evaluate $g'(1)$. Explain reasoning.

(d) On the interval $(-4, 10)$ where is g increasing? Decreasing?

(e) On what intervals is g concave up/down?

(f) Find the absolute minimum and maximum values of g on the interval $[-4, 10]$.



Note that we want actual function values – not merely the x-locations where they occur. [Hint: Do you know the critical points, end points? Can you compute g at those points?]