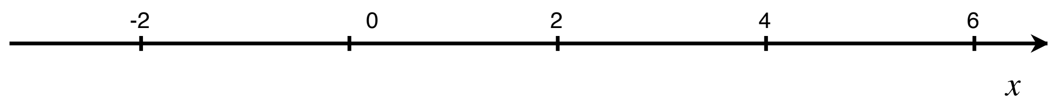
Worksheet

To learn the conceptual foundations of integration, it is helpful get comfortable with the indexing and notation that is commonly used. There is no science (or math!) to any of this, and it is mainly an exercise in “bookkeeping.” But, it is still important to learn this, so that integration concepts are easier to understand.

Consider the sketch of the x-axis shown below



Suppose we want to evaluate some function $f(x)$ at a bunch of points in the interval $[-2, 4]$ and make a table of x and $f(x)$ values. For illustration, let $f(x) = 2x - 1$ and suppose we evaluate it at 4 equally spaced points in $[-2, 4]$ as shown in this table

i (index)	x	$f(x)$
0	$x_0 = -2$	$f(x_0) = -5$
1	$x_1 = 0$	$f(x_1) = -1$
2	$x_2 = 2$	$f(x_2) = 3$
3	$x_3 = 4$	$f(x_3) = 7$

Notice that we can express the x and $f(x)$ values completetly in terms of the index i:

$$x_i = -2 + 2i, \qquad f(x_i) = 2(-2 + 2i) - 1 = -5 + 4i$$

Don’t just take my word for it! Plug in $i = 0, 1, 2, \ldots$ and see if you get the values shown in the table.

Exercise:

(a) Make a similar table and evaluate $f(x)$ at 7 equally spaced points in $[-2, 4]$. Write expressions for x_i, $f(x_i)$ in terms of the index i.

(b) Repeat with 13 equally spaced points.

(c) Repeat with $(n+1)$ equally spaced points. This time your x_i, $f(x_i)$ expressions will also contain n.