MATH 120: Makeup Quiz - 4/25/2019

A government report on housing costs says that single-family home prices nationwide are skewed to the right, with a mean of \$265,700. We want to see how home prices in Indiana compare with this figure. We collect data on a random sample of 43 homes for sale in Indiana and find a mean price of \$243,300 with standard deviation \$53,600.

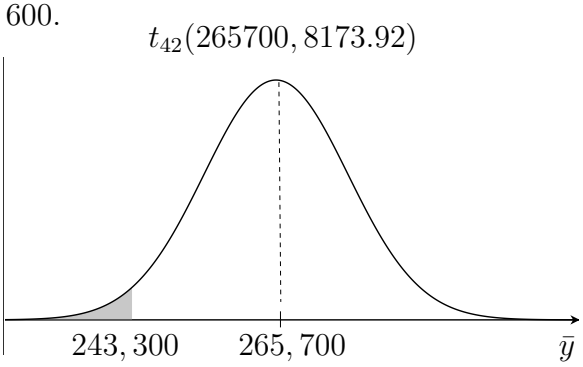
(I) [5 points] Carry out a hypothesis test to determine whether housing costs in Indiana are lower than the nationwide figures. You may omit checking the conditions. Be sure to show all other steps, including hypotheses, model, computations and conclusion.

(II) [1 point] Suppose we were to compute a confidence interval to answer the same question. What confidence level would match the significance level you chose for the hypothesis test? NOTE: You are not being asked to compute the confidence interval here – only what confidence level would be appropriate and why.

Solution

(I) Let μ = true mean price of single-family homes in Indiana,
I will use a significance level of 10%; i.e., $\alpha = 0.1$.

* Null hypothesis $H_0 : \mu = \$265,700$
  Alt. hypothesis $H_A : \mu < \$265,700$ (1 tail since we want to know if it is lower)
* Omit conditions check, as instructed.
* Sampling distribution model (based on H_0) is: $t_{42}(265,700, \frac{53,600}{\sqrt{43}})$:
  Sample info: $n = 43$, $\bar{y} = \$243,300$, $s = \$53,600$.
  $$\text{SE} = \frac{s}{\sqrt{n}} = \frac{53,600}{\sqrt{43}} = 8173.92$$
  $$t\text{-score} = \frac{\bar{y} - \mu}{\text{SE}} = \frac{243,300 - 265,700}{8173.92} = -2.74$$
  Want to find the area shown in the sketch, which corresponds to $t < -2.74$.



* From t-table, the closest lower *df* is 40. Our t-score has larger magnitude than the largest one given (i.e., 2.704). Therefore, our P-value is < 0.005 or 0.5%.
* Conclusion: This P-value is below our 10% significance level. Thus, we reject the null hypothesis, and infer that the prices of single-family homes in Indiana are statistically significantly lower than the nationwide average figures.

(II) A 1-tailed hypotheis test at a 10% significance level matches an 80% confidence level. This is because confidence intervals always have 2 tails, and we must allow 10% in each tail.

Grading: Total points possible = 6.

5 pt for (I): 0.5pt+0.5pt=correct hypotheses + clarify parameter used in them.
1pt=compute correct SE.
1pt=correct *df* and sampling distribution model $t_{42}(265700, 8173.92)$.
1pt=compute correct t-score.
1pt=correct P-value and conclusion.

1 pt for (II): no partial credit.