MATH 120: Quiz 7 - 4/04/2019

Based on past experience, a bank believes that 7% of customers who receive loans will not make payments on time. The bank selects a random sample of 300 loan customers from their database.

(a) Determine whether the conditions for applying the Central Limit Theorem are satisfied for modeling the sampling distribution of the proportion of customers who will not make payments on time.

(b) Find the probability that over 9% of this sample of 300 will not make timely payments.

[*As always, show steps and relevant sketches.*]

Solution

In this problem we have: $p = 0.07$ and $n = 300$

(a) Checking the conditions of the Central Limit Theorem:

(i) Is the sample independent?

Is it random: Yes. The question says random sample of 300 loan customers.

Is $n < 10\%$: Yes, if we assume the bank has at least 3000 loan customers, 300 would be less than 10%.

Since it is a random sample and $n < 10\%$, it is reasonable to assume independence.

(ii) Large enough sample?

Yes, since $np = 21$ and $n(1-p) = 279$, both are > 10.

All the conditions are satisfied. So we can apply the Central Limit Theorem.

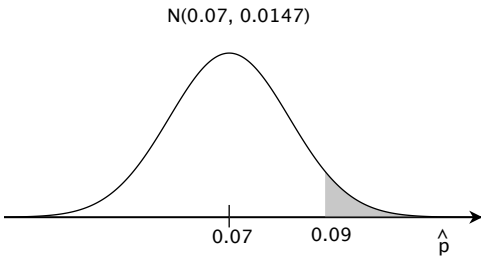
(b) According to the CLT, the sampling distribution follows the normal model $N(0.07, \sqrt{\frac{0.07 \cdot 0.93}{300}})$.

The sketch below shows $N(0.07, 0.0147)$, together with the area we want to find.

To find the shaded area:

Compute z-score: $z = \dfrac{0.09 - 0.07}{0.0147} = 1.358$

From the standard normal tables,
$z = 1.358$ corresponds to an area of 0.9127.
Thus, the shaded area $= 1 - 0.9127 = 0.0873$



Answer: The probability that over 9% will not make timely payments is 0.0873.

Grading: Total points possible = 6.

3 pt for (a): 1pt = identify n and p correctly.
1pt = correctly check independence condition.
1pt = correct check of large enough condition.

3 pt for (b): 0.5pt = sketch of model, showing what is to be computed.
1pt = correct normal model, with the right mean and SD.
1pt = compute correct z-score.
0.5pt = look up normal table correctly and get the required area.