Makeup Quiz - 12/06/2019

(I) Evaluate $\displaystyle\int_1^4 \frac{x^2-1}{\sqrt{x}}dx$.

(II) Let $G(x) = \displaystyle\int_x^2 \cos\sqrt{t}\,dt$. Find $G'(x)$. Be sure to justify your steps.

Solution

(I) Since this function is continuous on the given domain $[1,4]$, we can use the evaluation theorem. To find an antiderivative, simplify first:

$$\frac{x^2-1}{\sqrt{x}} = \frac{x^2}{\sqrt{x}} - \frac{1}{\sqrt{x}} = x^{3/2} - x^{-1/2}$$

Then

$$\int_1^4 \frac{x^2-1}{\sqrt{x}}\,dx = \int_1^4 (x^{3/2} - x^{-1/2})\,dx = \left[\frac{x^{5/2}}{5/2} - \frac{x^{1/2}}{1/2}\right]_1^4$$

$$= \left[\frac{2}{5}(4)^{5/2} - 2(4)^{1/2}\right] - \left[\frac{2}{5}(1)^{5/2} - 2(1)^{1/2}\right] = \frac{52}{5}$$

Answer: $\boxed{\displaystyle\int_1^4 \frac{x^2-1}{\sqrt{x}}\,dx = \frac{52}{5}}$

(II) Since $\cos\sqrt{t}$ is defined for $t \geq 0$, and it is also continuous, we can apply the Fundamental Theorem of Calculus when $t \geq 0$.

Using the interval peoperties of integrals, we have

$$G(x) = \int_x^2 \cos\sqrt{t}\,dt = -\int_2^x \cos\sqrt{t}\,dt$$

Thus, by the FTC

$$G'(x) = \frac{d}{dx}\left(-\int_2^x \cos\sqrt{t}\,dt\right) = -\cos\sqrt{x}$$

Answer: $\boxed{G'(x) = -\cos\sqrt{x}}$

Grading: Total points possible = 6.

4 pt for (I): 1 pt = correctly simplify to powers of x.
2 pt = get correct antiderivative.
1 pt = plugin bounds correcly and get answer.

2 pt for (II): 1pt+1pt = correct answer + correct steps/reason.