Quiz 8 - 11/08/2019

(I) This is an extension of exercise 29, Sec. 4.2, on this week's homework: Find the absolute minimum and maximum values of $g(y) = \dfrac{y-1}{y^2-y+1}$ on the interval $[-1, 3]$. Be sure to show steps, and state both y and $g(y)$ values in your answers.

(II) Indicate true or false for each of the following, with brief justification:
(a) If f has a local minimum at $x = c$ then $f'(c) = 0$.
(b) If f has an absolute minimum at $x = c$ then $f'(c) = 0$.

Solution

(I) Start by finding critical points:

$$g(y) = \frac{y-1}{y^2-y+1} \Rightarrow g'(y) = \frac{(y^2-y+1)\cdot(1)-(y-1)\cdot(2y-1)}{(y^2-y+1)^2}$$

$$= \frac{(y^2-y+1)-(2y^2-3y+1)}{(y^2-y+1)^2} = \frac{-y^2+2y}{(y^2-y+1)^2}$$

For C.P, we want $g'(y) = 0 \Rightarrow$ make the numerator 0, while denominator $\neq 0$.

$2y - y^2 = 0 \Rightarrow y(2-y) = 0$. So, $y = 0$ or $y = 2$.

Check denominator: $(0^2 - 0 + 1)^2 \neq 0$; $(2^2 - 2 + 1)^2 \neq 0$.

Thus, we have 2 critical points: $y = 0$, $y = 2$

Both C.P.'s are in the interval $[-1, 3]$. Evaluate g at the C.P.'s and end points:

y	-1	0	2	3
$g(y)$	$-2/3$	-1	$1/3$	0.286

Answer: **Absolute minimum: $(0, -1)$. Absolute maximum: $(2, 1/3)$.**

(II) (a) $x = c$ must be a critical point if f has a local minimum there. That means $f'(c) = 0$ or $f'(c) =$DNE. Thus, the correct answer here is "false." But, any answer that reflects the correct understanding in the reasoning will get partial credit.

(b) False. Absolute extremes need not occur at critical points. They can occur at end-points as well. Thus, $f'(c)$ can have any value.

Grading: Total points possible = 6.

4 pt for (I): 1 pt = correct derivative of g.
1 pt = find two correct C.P.s.
2 pt = tabulate values at C.P.s & end points, and state correct answers.

2 pt for (II): 1 pt each: 0.5 + 0.5 for answer + reason.