

Quiz 6 - 10/18/2019

- (a) Find dy/dx and simplify: $\sin x + \cos y = \sin x \cos y$.
(b) Find an equation of the tangent line to the graph of the above curve at $(\pi, \pi/2)$.

Solution

- (a) Differentiate each term with respect to x .

Term 1: $\frac{d}{dx}(\sin x) = \cos x$

Term 2: $\frac{d}{dx}(\cos y) = -\sin y \frac{dy}{dx}$

Term 3: $\frac{d}{dx}(\sin x \cos y) = (\sin x) \frac{d}{dx}(\cos y) + (\cos y) \frac{d}{dx}(\sin x)$
 $= (\sin x)(-\sin y) \frac{dy}{dx} + (\cos y)(\cos x)$

Put everything together and solve for dy/dx :

$$\cos x - \sin y \frac{dy}{dx} = -\sin x \sin y \frac{dy}{dx} + \cos x \cos y$$

$$\Rightarrow \sin y \frac{dy}{dx} (\sin x - 1) = \cos x (\cos y - 1)$$

$$\therefore \frac{dy}{dx} = \frac{\cos x (\cos y - 1)}{\sin y (\sin x - 1)}$$

- (b) Equation of tangent line at $(\pi, \pi/2)$:

We want $y = mx + b$, where $m = \frac{dy}{dx}$ at $(\pi, \pi/2)$

$$\text{Thus, } m = \frac{\cos(\pi)(\cos(\pi/2) - 1)}{\sin(\pi/2)(\sin(\pi) - 1)} = \frac{-1(0 - 1)}{1(0 - 1)} = -1$$

Equation of tangent: $y = -x + b$.

Plugin $(\pi, \pi/2)$ to solve for b : $\pi/2 = -\pi + b \Rightarrow b = 3\pi/2$.

Answer: $y = -x + 3\pi/2$

Grading: Total points possible = 6.

- 4 pt for (a): 0.5 pt = correct derivative of Term 1.
1 pt = correctly differentiate Term 2.
1.5 pt = correctly differentiate Term 3.
1 pt = correctly solve for dy/dx .
2 pt for (b): 1 pt = find correct slope of tangent line.
1 pt = find intercept and put together correct equation.