Quiz 5 - 10/11/2019

(I) Let $f(x) = \sin(x) - \cos(x)$. Find all the x values where the graph of f has horizontal tangents.

(II) If f is a differentiable function, find an expression for the derivative of $y = \dfrac{f(x)}{\sqrt{x}}$.

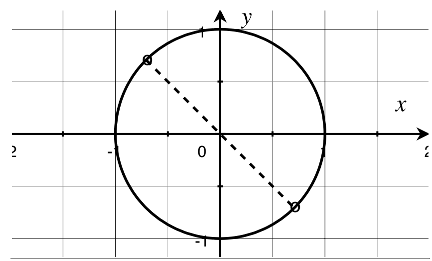
As always, show steps and reasoning for all solutions.

Solution

(I) The graph of f has horizontal tangents when $f'(x) = 0$.
Here we have: $f'(x) = \cos(x) + \sin(x)$.

$f'(x) = 0$ implies $\cos(x) + \sin(x) = 0$

Thus, we want: $\sin(x) = -\cos(x)$.



From the unit circle we see $\sin\theta = -\cos\theta$ at 2 places:
$\theta = 180° - 45° = 135°$, and $\theta = 360° - 45° = 315°$.

In radians: $135° = 3\pi/4$ and $315° = 7\pi/4$.

f has horizontal tangents at: $\boxed{x = 3\pi/4 + n\pi}$ where $n = 0, \pm 1, \pm 2, \ldots$

(II) Given: $y = \dfrac{f(x)}{\sqrt{x}}$.

Using quotient rule: $y' = \dfrac{\sqrt{x} \cdot f'(x) - f(x)(\sqrt{x})'}{(\sqrt{x})^2}$

$$= \frac{\sqrt{x} \cdot f'(x) - f(x) \cdot \frac{1}{2\sqrt{x}}}{x}$$

Multiply numerator & denom by $2\sqrt{x}$ to simplify and get:

$$= \frac{2x \cdot f'(x) - f(x)}{2x\sqrt{x}}$$

Answer: $\boxed{y' = \dfrac{2xf'(x) - f(x)}{2x\sqrt{x}} \quad \text{OR} \quad \dfrac{f'(x)}{\sqrt{x}} - \dfrac{f(x)}{2x\sqrt{x}}}$

Grading: Total points possible = 6.

3 pt for (I): 0.5 pt = know/show we want $f' = 0$.
1.5 pt = find f' and show we want $\sin(x) = -\cos(x)$ or $\tan(x) = -1$.
1 pt = solve correctly for x-values (including $n\pi$).

3 pt for (II): 0.5 pt = correctly set up QR formula.
1.5 pt = correctly differentiate numerator terms.
1 pt = simplify.