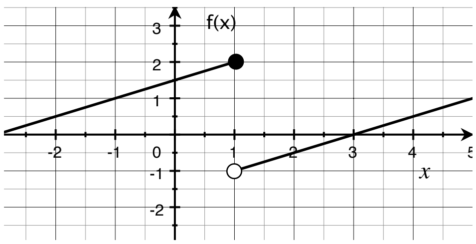
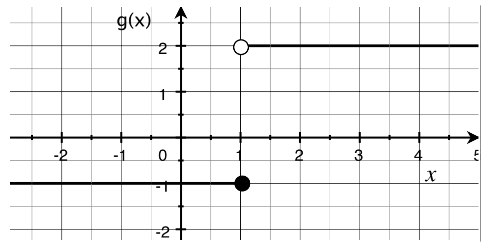
Quiz 3 - 9/13/2019

(I) Evaluate the following limits using the graphs of f and g given below (must include correct/valid mathematical steps – no credit without it):

(a) $\lim_{x\to 1}[f(x)+g(x)]$ (b) $\lim_{x\to -1}[f(x)\cdot g(x)]$

Hint: Be careful!





(II) Use the mathematical definition of continuity to determine whether the following function is continuous at $x = 0$ (show steps)

$$f(x) = \begin{cases} \dfrac{\sqrt{1+x}-1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Solution

(I) According to limit laws: Limit of a sum = sum of the limits, provided both limits individually exist.

(a) When $x \to 1$, we cannot apply the limit laws directly because neither limit exists. However, both functions have left- and right- limits. So, we can apply limit laws to find the limit on each side separately:

$\lim_{x\to 1^-}[f(x)+g(x)] = \lim_{x\to 1^-} f(x) + \lim_{x\to 1^-} g(x) = 2 - 1 = 1$

$\lim_{x\to 1^+}[f(x)+g(x)] = \lim_{x\to 1^+} f(x) + \lim_{x\to 1^+} g(x) = -1 + 2 = 1$

Since the left- and right- limit are the same, $\boxed{\lim_{x\to 1}[f(x)+g(x)] = 1}$

(b) For this case we can apply limit laws directly, since both limits exist:

$\lim_{x\to -1}[f(x)\cdot g(x)] = \lim_{x\to -1} f(x) \cdot \lim_{x\to -1} g(x) = 1\cdot(-1) = \boxed{= -1}$

(II) According to the definition, continuity at $x = 0$ requires: $\lim_{x\to 0} f(x) = f(0)$.

In this problem, $f(0) = 0$.

To find $\lim_{x\to 0} f(x)$ we must try to do some algebra and cancel the x in the denominator

$$\frac{\sqrt{1+x}-1}{x} = \frac{(\sqrt{1+x}-1)\cdot(\sqrt{1+x}+1)}{x\cdot(\sqrt{1+x}+1)} = \frac{x}{x\cdot(\sqrt{1+x}+1)} = \frac{1}{\sqrt{1+x}+1} \quad (\text{provided } x \neq 0)$$

Therefore, $\lim_{x\to 0} f(x) = \lim_{x\to 0}\left[\dfrac{1}{\sqrt{1+x}+1}\right] = \dfrac{1}{2}$

Since $f(0) = 0$ and $\lim_{x\to 0} f(x) = 1/2$, it follows that f is not continuous at $x = 0$.

Grading: Total points possible = 6.

3 pt for (I):	1.5pt for (a) + 1.5 pt for (b).
	No credit for correct answers without correct reason.
3 pt for (II):	0.5 pt = Attempt to apply correct defn of continuity at $x = 0$.
	0.5pt = Find correct $f(0)$.
	1.5pt = Find correct $\lim_{x \to 0} f(x)$.
	0.5 pt = State correct conclusion.