# Teaching with a Smile 

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## Outline

(1) NP-Complete Puzzles
(2) Combinatorics
(3) Other Topics

## Puzzles

## Examples

Sudoku, Calcudoku (Kenken), Kakuro, Hidato, Nurikabe, Parks, Snail, Skyscrapers, etc.

## Benefits

- Show students mathematics is much broader and can be much more fun than they thought
- They'll pay attention better throughout the whole class
- Allow students to participate in class more comfortably
- Introduce students to P vs. NP Millenium Prize question
- Can connect this to limits and o, O-notation


## Other Fun Stuff

Puzzles are enough to engage students and open their minds about math a little, but we can do more by introducing other topics, combinatorics for one.

## Combinatorics Part 1: Jumbles

## ACNIP

## Combinatorics Part 1: Jumbles

## Combinatorics Part 1: Jumbles

## YNUFN

## Combinatorics Part 1: Jumbles

## Combinatorics Part 1: Jumbles

## DELONO

## Combinatorics Part 1: Jumbles

## Combinatorics Part 1: Jumbles

## MAGAZIN

## Combinatorics Part 1: Jumbles

## AMAZING

## Combinatorics Part 1: Jumbles

## ENJOYUR

## Combinatorics Part 1: Jumbles

## JOURNEY

## Combinatorics Part 2: Formula

Number of permutations of a word $=$

## (length of word)!

$\overline{\prod_{\text {letter=A }}^{Z}(\text { number of times letter appears in word)! }}$

## Combinatorics Part 3: Permutations

$$
\binom{n}{r}=(\text { number of permutations of } \underbrace{\mathrm{AA} \ldots \mathrm{~A}}_{r} \underbrace{\mathrm{BB} \ldots \mathrm{~B}}_{n-r})=\frac{n!}{r!(n-r)!}
$$

(label the objects $1, \ldots, n$ and label the chosen ones with $A$ 's and the rest with $B^{\prime} \mathrm{s}$ )

## Combinatorics Part 4: Pascal's Triangle

How many ways to get from the top ? to the bottom ones without going up?


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## Combinatorics Part 5: Gathering What We've Learned

The number of ways to get to the $r$-th spot in the $n$-th row ( $n$ and $r$ start at 0 ) is the number of permutations of

$$
\underbrace{\mathrm{RR} \ldots \mathrm{R}}_{r} \underbrace{\mathrm{LL} \ldots \mathrm{~L}}_{n-r}
$$

which is

$$
\frac{n!}{r!(n-r)!}=\binom{n}{r}
$$

## Combinatorics Part 5: Gathering What We've Learned

We've proven combinatorially that

$$
\binom{n}{r}+\binom{n}{r+1}=\binom{n+1}{r+1}
$$

Exercise. Prove it algebraically!

## Combinatorics Part 6: Binomial Coefficients

$$
(x+y)^{n}=\sum_{r=0}^{n}\binom{n}{r} x^{r} y^{n-r}
$$

because the number of $x^{r} y^{n-r}$ terms in the expansion is the number of permutations of

$$
\underbrace{x x \cdots x}_{r} \underbrace{y y \cdots y}_{n-r}
$$

## Combinatorics Part 7: Application to Calculus

Let $u$ and $v$ be functions. The $n$-th derivative of $u v$ is:

$$
\begin{gathered}
(u v)^{(n)}=\sum_{r=0}^{n}\binom{n}{r} u^{(r)} y^{(n-r)} \\
\left(x e^{x}\right)^{(n)}=\binom{n}{0} x^{(0)}\left(e^{x}\right)^{(n)}+\binom{n}{1} x^{(1)}\left(e^{x}\right)^{(n-1)}=x e^{x}+n e^{x}
\end{gathered}
$$

Later find the Taylor series for $x e^{x}$ centered at 2.

## List of Other Topics

## Topics

- Induction
- Rational / Irrational Numbers
- Magic squares
- Golden ratio
- Modular Arithmetic
- MANY more


## Induction

- Prove the formulas used for Riemann sums:

$$
\sum_{k=1}^{n} k=\frac{n(n+1)}{2}, \sum_{k=1}^{n} k^{2}=\frac{n(n+1)(2 n+1)}{6}, \sum_{k=1}^{n} k^{3}=\frac{n^{2}(n+1)^{2}}{4}
$$

- Fibonacci numbers
- Find the $n$-th derivative of a function, plug in $c$ to get Taylor series centered at $c$


## More Induction

- The number of times it takes to break up a chocolate bar with $n$ pieces is $n-1$.
- You can tile a $2^{n} \times 2^{n}$ chessboard with one tile removed by using only L-trominoes
- Games like Nim


## Rational / Irrational Numbers

## Question

Is an irrational power of an irrational number always irrational?
Hint: $(\sqrt{2})^{2}=2$.

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## Answer

No: $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}=2$.

## Caution: Don't Get Overzealous!

## Don't feed your students a whole chicken with

 Thanksgiving dinner! I.e. don't do too many puzzles!
## Attitude

## Easy Problems

Easy problems are there for students to practice and feel good about their skill set

## Hard Problems

Hard problems are the ones they don't know how to solve immediately. These are the fun ones where they get to be creative and discover something new for themselves.

Thank you so much for your attention!

