Cohomology of Topological Groups and Metrizability

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Abstract

Let G be a topological group and A a topological G-module. Suppose we are interested in the theory of cohomology groups $H^n(G, A)$ but we want to consider the topological data of G and A. How many ways are there to make such a theory? Lots! We will go over the usual equivalent definitions of group cohomology and talk about generalizing them to this setting, how the theory falls apart into many pieces, and some attempts at putting the pieces back together, one of which involves pseudometrizability and complete metrizability of A. No prerequisites are assumed.

1 Introduction

For an abstract group G, there is one "canonical" cohomology theory $(H^n(G, A))_{n=0}^{\infty}$ for G-modules A. It is well-known that the theory of group cohomology has many applications in number theory, group theory, and beyond. Now suppose we have a topological group G and a topological G-module A (that is, a G-module that is an abelian topological group such that the map $G \times A \to A$ is continuous). We could consider the topological data of G and A and hope to obtain more detailed information from the groups $H^n(G, A)$. Indeed, this has found applications in probability theory [Gui73], ergodic theory [AM13], and (most interesting to me) number theory (see [Lic09], [Fla08], [KR12]). It turns out that the many definitions of the usual group cohomology have strictly different generalizations when involving the topologies of G and A, so we might get different results depending on which definition we use.

It may seem that the ultimate goal is then to find a definition of cohomology for topological groups that inherits all the nice properties from group cohomology. But after more than 50 years of trying to find such a nice definition, no one has completely succeeded (though some have certainly come close) without restricting G and A. Indeed, it may be more reasonable to develop a variety of cohomology theories which are useful in their own ways. The purpose of this talk is to list some of the plausible definitions for the cohomology of topological groups and mention a few ways to restrict G and A so that some of the definitions coincide.

Some other good theories for the cohomology of topological groups not mentioned here are [HM62], [FW12], [Seg70], and [Sta78].

2 Usual Definitions of Group Cohomology

Definitions 1 and 2. For an abstract group G and a G-module A, we define $H^n(G, A) = \mathrm{Ext}^n(\mathbb{Z}, A)$, where the Ext's are taken in the category G-mod of G-modules and $\mathbb{Z}$ has trivial G-action. Equivalently, $H^n(G, -)$ is the n-th right-derived functor of the functor from G-mod to Ab (the category of abelian groups) taking A to A^G, the group of elements of A that are fixed by G. Note that $\mathrm{Hom}_{G\text{-mod}}(\mathbb{Z}, A) = A^G$ and $\mathrm{Ext}^n(\mathbb{Z}, -)$ are the derived functors of $\mathrm{Hom}(\mathbb{Z}, -)$; this is why the two definitions are equivalent.

Definition 3. There is a third equivalent definition, using inhomogeneous cochains (see the appendix for the definition using homogeneous cochains instead). Consider the complex $(C^n(G, A), \delta_n : C^n(G, A) \to C^{n+1}(G, A))_{n=0}^{\infty}$, where $C^n(G, A)$ is the set of all maps $G^n \to A$ (we treat G^0 as a point so $C^0(G, A) = A$) and

$$\begin{aligned}(\delta_n(f))(x_0, \dots, x_n) &= x_0 f(x_1, \dots, x_n) \\ &+ \sum_{k=1}^{n} (-1)^k f(x_0, \dots, x_{k-2}, x_{k-1}x_k, x_{k+1}, \dots, x_n) + f(x_0, \dots, x_{n-1})\end{aligned} \tag{2.1}$$

The elements of $C^n(G, A)$ are called inhomogeneous cochains. $H^n(G, A)$ is the cohomology of this complex.

Definition 4. The last familiar equivalent definition is the singular cohomology $H^n(B_G, A)$ of the classifying space B_G of the discrete group G with coefficients in A. Of course, if G is discrete, then this space is an Eilenberg-Maclane space, i.e. a $K(G, 1)$. Indeed, one way to generalize the theory to topological groups G is to consider the cohomology $H^n(B_G, A)$ of the classifying space B_G for the topological group G, though this cannot take into account the topology on A.

3 Wigner's Cohomology

David Wigner [Wig73] invented a cohomology theory $H^n_W(G, A)$ for topological groups which generalizes the cohomology $H^n(B_G, A)$ of the classifying space B_G, that is, for discrete topological G-modules A we have $H^n_W(G, A) = H^n(B_G, A)$[1]. Here is how Wigner's cohomology is defined.

We first construct a semisimplicial G-space $S(G)$, i.e. a semisimplicial object in the category of G-spaces and continuous G-maps. The n-simplex S_n of $S(G)$ is $G \times G^n$, where G acts on $G \times G^n$ by left multiplication on the first coordinate. The face maps $d_i : S_n \to S_{n-1}$ are given by

$$d_i(g_0, g_1, \dots, g_n) = \begin{cases} (g_0, g_1, \dots, g_i g_{i+1}, \dots, g_n), & 0 \le i < n \\ (g_0, \dots, g_{n-1}), & i = n \end{cases}$$

The degeneracy maps $s_i : S_n \to S_{n+1}$ are given by

$$s_i(g_0, \dots, g_n) = (g_0, \dots, g_{i-1}, 1, g_i, \dots, g_n)$$

Consider the natural projections p_n from $S_n \times A$ to S_n/G, which is isomorphic to the topological space G^n with trivial G-action. The face maps and degeneracies of $S(G)$ induce faces and degeneracies on the spaces $S_n \times A$ and on the spaces S_n/G, producing two more semisimplicial spaces, and these face maps and degeneracies commute with the p_n. Let T_n be the sheaf of germs of continuous sections of p_n. Since $g \cdot 0 = 0 \in A$ for all $g \in G$, T_n is isomorphic to the sheaf of germs of continuous A-valued functions on

[1] More precisely, if A is a discrete G-module, then $H^n_W(G, A)$ is the sheaf cohomology of B_G with coefficients in the locally constant sheaf A.

S_n/G. The T_n have face maps and degeneracies induced by those of $S(G)$ and thus form a semisimplicial sheaf $T(G,A)$ over the S_n/G.

Now apply the (second) canonical semisimplicial resolution functor to $T(G,A)$ to get canonical flabby resolutions for each T_n, and thus a double complex of groups of global sections. Let us explain. Note that the usual canonical resolution of a sheaf F is obtained by letting $C^0(F) = \prod_{x\in X}(i_x)_*F_x$, embedding F in $C^0(F)$, taking the quotient F_1, embedding F_1 in $C^0(F_1)$, etc. to get the resolution $0 \to F \to C^0(F) \to C^0(F_1) \to \cdots$. Here we mean instead the resolution $0 \to F \to C^0(F) \to C^0(C^0(F)) \to \cdots$. Denote by $0 \to T_n \to T_{1,n} \to T_{2,n} \to \cdots$ the flabby resolution of T_n. For each i and n we have a map $T_{n-1} \to (d_i)_*T_n$, hence maps $T_{j,n-1} \to (d_i)_*T_{j,n}$, and hence maps $\Gamma(S^{n-1}/G, T_{j,n-1}) \to \Gamma(S^n/G, T_{j,n})$. By taking the alternating sum of these maps over i we get a map $\Gamma(S^{n-1}/G, T_{j,n-1}) \to \Gamma(S^n/G, T_{j,n})$, hence a double complex. $H^n_W(G,A)$ is the cohomology of this double complex.

4 Cochain Definitions

To generalize definition 3 of section 2 to incorporate the topological data of G and A we can set $C^n(G,A)$ to be the set of

1. continuous maps $G^n \to A$ [Hu52],
2. measurable[2] maps $G^n \to A$ [Moo76], or
3. locally continuous measurable maps $f : G^n \to A$, i.e. those f which are measurable, satisfy $f(1,1,\ldots,1) = 0 \in A$, and such that $f|U$ is continuous for some neighborhood U of $(1,1,\ldots,1) \in G^n$ [KR12].

We define the inhomogeneous coboundary operator δ_n just as before by equation (2.1). Taking cohomology of the resulting complexes gives cohomology theories

1. $H^n_c(G,A)$, called the **continuous cochain theory**;
2. $H^n_m(G,A)$, called the **measurable cochain theory**;

[2]A map $f : X \to Y$ of topological spaces is (Borel-)measurable if the preimage $f^{-1}(U)$ of every measurable set U in Y is in the Borel σ-algebra of X, the σ-algebra generated by the open sets in X.

3. $H^n_{lcm}(G,A)$, called the **locally continuous measurable cochain theory**

respectively, and these are in general not the same, but

$$H^n_c(G,A) \subseteq H^n_{lcm}(G,A) \subseteq H^n_m(G,A)$$

Theorem 4.1. *Consider the short exact sequence of topological G-modules*

$$0 \to A \to B \xrightarrow{\phi} C \to 0$$

If ϕ has a continuous global section (respectively, a measurable section, or a locally continuous measurable section) then there is a long exact sequence on cohomology

$$\cdots \to H^n(G,A) \to H^n(G,B) \to H^n(G,C) \to H^{n+1}(G,A) \to \cdots$$

for the continuous (respectively, measurable, or locally continuous measurable) cochain theory.

Example computation. If G is connected and A is discrete, then $C^n_c(G,A)$ consists of constant maps, so $H^n_c(G,A) = 0$ for $n > 0$. We leave it as an easy exercise to the reader to show that the action of G on A must in fact be trivial in this case, using the continuity of $G \times A \to A$. Now consider the exact sequence of trivial $\mathbb{S}^1$-modules

$$0 \to \mathbb{Z} \to \mathbb{R} \xrightarrow{\pi} \mathbb{S}^1 \to 0 \tag{4.1}$$

By the above remark, $H^1(\mathbb{S}^1,\mathbb{Z}) = H^2(\mathbb{S}^1,\mathbb{Z}) = 0$, and $H^1(\mathbb{S}^1,\mathbb{R}) = \mathrm{Hom}_{\mathrm{cont}}(\mathbb{S}^1,\mathbb{R}) = 0$ because there are no nontrivial continuous homomorphisms from $\mathbb{S}^1$ to $\mathbb{R}$. On the other hand, $H^1(\mathbb{S}^1,\mathbb{S}^1) = \mathrm{Hom}_{\mathrm{cont}}(\mathbb{S}^1,\mathbb{S}^1) = \mathbb{Z} \neq 0$. Thus the sequence

$$H^1(\mathbb{S}^1,\mathbb{Z}) \to H^1(\mathbb{S}^1,\mathbb{R}) \to H^1(\mathbb{S}^1,\mathbb{S}^1) \to H^2(\mathbb{S}^1,\mathbb{Z})$$

is not exact. Note that π does not have a continuous global section, even though it has local sections everywhere. If π did have a continuous global section, then we would indeed have a long exact sequence on cohomology.

It should be noted that if we work in the category $\mathcal{M}^p_G$ of Polish (that is, complete metric second-countable) G-modules, then any epimorphism $B \xrightarrow{\phi} C$ has a measurable section. One can also topologize the groups $H^n_m(G,A)$, and this is part of what makes Moore's measurable cochain theory so attractive.

Theorem 4.2 ([Wig73]). *If G is locally compact, σ-compact*[3]*, and zero-dimensional*[4]*, and A is Polish, then $H_c^n(G,A) = H_m^n(G,A)$ for all n.*

Application. In class field theory, one computes the cohomology of profinite groups G, with the profinite topology, by using $H_c^n(G,A)$ for discrete G-modules A. Note that a topological group is profinite if and only if it is compact and totally disconnected. In particular, theorem 4.2 applies to this situation (as long as A is countable).

5 $H^2(G,A)$

It is well-known that, for the standard theory of group cohomology, given a G-module A, $H^2(G,A)$ classifies the extensions

$$1 \to A \to E \xrightarrow{\psi} G \to 1 \tag{5.1}$$

of abstract groups with the fixed action of G (i.e. such that the action of G on A by conjugation is the same as the original action of G on A given in the definition of A as a G-module). Note that this does not give an action of G on E, so this is not an extension of G-sets.

Theorem 5.1. *[Hu52] Given a topological G-module A, $H_c^2(G,A)$ classifies the extensions (5.1) of topological groups with the fixed action of G on A such that the map ψ has a continuous section.*

Theorem 5.2. *[Moo76] If G is locally compact, Hausdorff, and second countable, and A is a second countable topological G-module whose topology is given by a complete metric, then $H_m^2(G,A)$ classifies all extensions (5.1) of topological groups with the fixed action of G on A.*

Moore used this theorem as part of the justification that his cohomology $H_m^n(G,A)$ may be the “right” generalization of the usual group cohomology theory to topological groups and G-modules.

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[3] A space is σ-compact if it is the union of countably many compact subsets.

[4] The concept of dimension here is that of Lebesgue dimension: we say that for a space X, $\dim X \leq n$ if any covering $\{U_i\}$ of X has a refinement $\{V_i\}$ such that for all $x \in X$, $x \in V_i$ for at most $n+1$ indices i. The dimension m of X is the smallest m such that $\dim X \leq m$ but not $\dim X \leq m-1$ (if no such m exist, then $\dim X = \infty$).

Theorem 5.3. *[KR12] With the assumptions in theorem 5.2, $H^2_{lcm}(G, A)$ classifies all extensions (5.1) of topological groups with the fixed action of G on A such that ψ has a local section.*

6 Yoneda Ext Definition

In this section we generalize the definition $H^n(G, A) = \mathrm{Ext}^n(\mathbb{Z}, A)$ to the topological setting. We first define quasi-abelian S-categories, as Yoneda did, and then use them to define cohomology theories for topological groups.

A morphism $f : A \to B$ in an additive category is **proper**, or **strict**, if the natural map $\mathrm{coker}(\ker f) \to \ker(\mathrm{coker}\, f)$ is an isomorphism and, in particular, these kernels and cokernels exist. In this case, the object associated to $\mathrm{coker}(\ker f)$ is called the **image**[5] $\mathrm{Im}(f)$ of f. A sequence $A \xrightarrow{f} B \xrightarrow{g} C$ is **exact** if f and g are proper, $g \circ f = 0$, and the natural map $\mathrm{Im}(f) \to \ker(g)$ is an isomorphism.

An S**-category** $(\mathcal{C}, S)$ is an *additive* category $\mathcal{C}$ together with a class S of morphisms of $\mathcal{C}$ such that:

(S1) All isomorphisms are in S, and all maps in S are proper.

(S2) For any two morphisms $f : A \to B, g : C \to D$ in S, the morphism $f \oplus g : A \oplus C \to B \oplus D$ is in S.

(S3) If $\phi \in S$ then $\ker \phi \in S$ and $\mathrm{coker}\, \phi \in S$.

(S4) Any $f \in S$ can be written $f = me$, where $m, e \in S, \ker m = 0, \mathrm{coker}\, e = 0$, and any such composition me is in S.

The class $P(\mathcal{C})$ of all proper morphisms satisfies all these properties and so is the largest possible class S for $\mathcal{C}$. An S-category is **quasi-abelian** if it satisfies the following four conditions.

(Q0) A composition of epimorphisms in S is an epimorphism in S.

(Q0*) A composition of monomorphisms in S is a monomorphism in S.

[5]Technically, this object is not unique, but only unique up to isomorphism.

(Q2) Every pullback of an epimorphism in S exists and is (an epimorphism) in S.

(Q2*) Every pushout of a monomorphism in S exists and is (a monomorphism) in S.

A category C is **quasi-abelian** if $(C, P(C))$ is quasi-abelian.

Primary example. The category $\mathcal{M}_G$ of all topological G-modules and continuous G-maps is quasi-abelian. In fact, every morphism $f : A \to B$ in $\mathcal{M}_G$ has a kernel and a cokernel, but f is proper if and only if the induced map $A \to \mathrm{Im}(A)$ is an open map, when $\mathrm{Im}(A)$ is considered as a subspace of B.

For any quasi-abelian S-category (C, S), we can define $\mathrm{Ext}^n_{C,S}(A, B)$ to be the class of extensions (exact sequences)

$$X : 0 \to B \to E_n \to \cdots \to E_1 \to A \to 0$$

modulo the equivalence relation generated by commutative diagrams

$$\begin{array}{ccccccccccccc} X: & 0 & \to & B & \to & E_n & \to & \cdots & \to & E_1 & \to & A & \to & 0 \\ & & & \| & & \downarrow & & & & \downarrow & & \| & & \\ X': & 0 & \to & B & \to & E'_n & \to & \cdots & \to & E'_1 & \to & A & \to & 0 \end{array}$$

It turns out that

1. $\mathrm{Ext}^n_{C,S}(A, B)$ is an abelian group (see [Yon60, p. 537] for more details.);

2. for any $Z \in C$, any short exact sequence $0 \to A \xrightarrow{f} B \xrightarrow{g} C \to 0$ in S (i.e. with $f, g \in S$) gives a long exact sequence

   $$0 \to \mathrm{Hom}_C(Z, A) \to \mathrm{Hom}_C(Z, B) \to \mathrm{Hom}_C(Z, C) \to \mathrm{Ext}^1_{C,S}(Z, A) \to \cdots$$

3. and there is a universality property: for any collection of functors $(h^n)_{n=0}^{\infty}$ from C to Ab and any natural transformation $\eta_0 : h^0 \to \mathrm{Hom}_C(Z, -)$, if any short exact sequence as above in S gives a long exact sequence

   $$h^0(A) \to h^0(B) \to h^0(C) \to h^1(A) \to \cdots$$

then there are unique natural transformations $\eta_n : \mathrm{Ext}^n_{C,S}(Z, -) \to h^n$ that extend η_0.[6]

For any class S of morphisms of $\mathcal{M}_G$ such that $(\mathcal{M}_G, S)$ is quasi-abelian, we can then define a cohomology theory by $H^n(G, A) = \mathrm{Ext}^n_{\mathcal{M}_G,S}(\mathbb{Z}, A)$, where $\mathbb{Z}$ has trivial G-action and discrete topology (whether this cohomology theory is useful or not then depends on the category C, the class S, the applicability of the theory, etc.).

There are classes S of morphisms in $\mathcal{M}_G$ corresponding to the cohomology theories $H^n_c(G, A), H^n_m(G, A), H^n_{lcm}(G, A)$ defined in section 4. These classes S are defined by insisting that a short exact sequence

$$0 \to A \to B \xrightarrow{\phi} C \to 0 \tag{6.1}$$

is in S if and only if ϕ has a section that is continuous, measurable, or locally continuous and measurable, respectively.

Theorem 6.1. *If G is locally compact, second countable, and Hausdorff, then* $\mathrm{Ext}^n_{\mathcal{M}^P_G, P(\mathcal{M}^P_G)}(\mathbb{Z}, A) = H^n_m(G, A)$.

Theorem 6.2. *If G is locally compact, then* $\mathrm{Ext}^n_{\mathcal{M}_G,S}(\mathbb{Z}, A) = H^n_c(G, A)$, *where S is the class of morphisms corresponding to the short exact sequences (6.1) where ϕ has a continuous section.*

At this point it is not known whether or not such a statement holds for $H^n_{lcm}(G, A)$.

7 Pseudometrics and Complete Metrics

In this section we will assume G is **weakly separable**[7], that is, for any open set U in G, the collection $\{gU \mid g \in G\}$ has a countable subcover.

[6] This means for any short exact sequence in C, the induced diagram of long exact sequences is commuative.

[7] This is the terminology used by Wigner and Lawrence Brown [Wig73]; it seems the more modern terminology is **trans-separable**, where "trans" stands for "translation" [Dre75]

Let $\mathcal{M}_G^p$ be the category of pseudometric G-modules, that is, those topological G-modules whose topology is induced by a pseudometric (a pseudometric d is "almost a metric" - it does not satisfy the condition that $d(x, y) = 0 \Rightarrow x = y$). Let $\mathcal{M}_G^{cm}$ be the category of topological G-modules whose topology is induced by a complete metric.

Theorem 7.1 (Lawrence Brown)**.** *If A and B are pseudometric G-modules, then $\mathrm{Ext}^n_{\mathcal{M}_G}(A, B) = \mathrm{Ext}^n_{\mathcal{M}_G^p}(A, B)$ for all n, that is, any extension of pseudometric G-modules by arbitrary G-modules is equivalent to an extension made up of only pseudometric G-modules.*

Proof. (Sketch). First consider the case $n = 1$. We have to show that if we have an extension

$$0 \to B \xrightarrow{\beta} E \xrightarrow{\alpha} A \to 0$$

where A and B are pseudometrizable, then E is pseudometrizable. We will show that, in fact, the topology on E is the sup of the two topologies T_A and T_B induced from the pseudometrics d_A on A and d_B on B, respectively. The topologies T_A and T_B are pseudometrizable, and the sup of two pseudometrizable topologies is also pseudometrizable (the pseudometric is given by the sum of the two given pseudometrics). The topology T_A is easy to describe: it is given by the pseudometric $d(x, y) = d_A(\alpha(x), \alpha(y))$. The hard part is describing T_B. We need the following beautiful theorem:

Theorem 7.2. *[Wil70, Theorem 12.2.3] A topological group G is pseudometrizable $\iff$ G is first-countable $\iff$ the topology of G is induced by a left-invariant pseudometric d (that is, $d(gx, gy) = d(x, y)$ for all $g, x, y \in G$).*

Using the fact that β is open onto its image (this is part of the definition of an exact sequence), we can find a countable collection of open sets in E whose intersections with B give a basis for B at $0 \in B$ and then use these open sets to get a topology T_B on E by translating them. The new topology will be first countable, hence pseudometrizable. We need G to be weakly separable so that the G-action on E is still continuous with the topology T_B.

In the case $n > 1$, we have an extension

$$0 \to B \xrightarrow{\beta} E_n \xrightarrow{\gamma_n} E_{n-1} \xrightarrow{\gamma_{n-1}} \cdots \to E_1 \xrightarrow{\alpha} A \to 0$$

where A and B are pseudometrizable but $E_1, \ldots, E_n$ are not necessarily. We construct a coarser, pseudometrizable, topology on E_n similarly to T_B

above. We then take the quotient Q_n of E_n with the new topology by B and consider the injection $Q_n \to E_{n-1}$. Now Q_n is pseudometric, so we can proceed as with E_n, replacing B by Q_n. We continue this process until we reach E_1, on which we construct the sup of the topologies from Q_2 and A as above. We thus get an extension of the same spaces, but now with a different topology and easily find that the canonical map from the old extension to the new one is a map in our category, hence the two are equivalent. $\square$

Theorem 7.3 (Lawrence Brown). *If A and B are complete metric G-modules, then $\mathrm{Ext}^n_{\mathcal{M}_G^{cm}}(A,B) = \mathrm{Ext}^n_{\mathcal{M}_G}(A,B)$ for all n, that is, any extension of even complete metric G-modules by arbitrary G-modules is equivalent to an extension made up of only complete metric G-modules.*

This theorem justifies working with the cohomology theories defined by $\mathrm{Ext}^n(\mathbb{Z}, A)$ in the category of complete metric G-modules as opposed to the larger category of all topological G-modules.

8 Appendix: Homogeneous Cochains

Instead of considering $C^n(G,A) = \mathrm{Map}(G^n, A)$, we consider $C^n_h(G,A) = \mathrm{Map}_G(G^{n+1}, A)$, the set of all G-equivariant maps $G^{n+1} \to A$ and the coboundary operator

$$\delta^h_n(f)(x_0, \ldots, x_n) = \sum_{k=0}^{n+1} (-1)^k f(x_0, \ldots, \hat{x}_k, \ldots, x_n)$$

When working with the continous, measurable, or locally continuous measurable cochain theory, $C^n(G,A)$ is defined by using continuous, measurable, or locally continuous measurable maps instead, respectively. The cohomology of the resulting complexes is the same as that of the complexes of inhomogeneous cochains.

References

[AM13] Tim Austin and Calvin C. Moore. Continuity properties of measurable group cohomology. *Math. Ann.*, 356(3):885–937, 2013.

[Dre75] L. Drewnowski. Another note on Kalton's theorems. *Studia Math.*, 52:233–237, 1974/75.

[Fla08] M. Flach. Cohomology of topological groups with applications to the Weil group. *Compos. Math.*, 144(3):633–656, 2008.

[FW12] Martin Fuchssteiner and Christoph Wockel. Topological group cohomology with loop contractible coefficients. *Topology Appl.*, 159(10-11):2627–2634, 2012.

[Gui73] A. Guichardet. Cohomology of topological groups and positive definite functions. *J. Multivariate Anal.*, 3:249–261, 1973.

[HM62] G. Hochschild and G. D. Mostow. Cohomology of Lie groups. *Illinois J. Math.*, 6:367–401, 1962.

[Hu52] Sze-tsen Hu. Cohomology theory in topological groups. *Michigan Math. J.*, 1:11–59, 1952.

[KR12] Arati S. Khedekar and C. S. Rajan. On cohomology theory for topological groups. *Proc. Indian Acad. Sci. Math. Sci.*, 122(2):221–242, 2012.

[Lic09] Stephen Lichtenbaum. The Weil-étale topology for number rings. *Ann. of Math. (2)*, 170(2):657–683, 2009.

[Moo76] Calvin C. Moore. Group extensions and cohomology for locally compact groups. III. *Trans. Amer. Math. Soc.*, 221(1):1–33, 1976.

[Seg70] Graeme Segal. Cohomology of topological groups. In *Symposia Mathematica, Vol. IV (INDAM, Rome, 1968/69)*, pages 377–387. Academic Press, London, 1970.

[Ser79] Jean-Pierre Serre. *Local fields*, volume 67 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1979. Translated from the French by Marvin Jay Greenberg.

[Sta78] James D. Stasheff. Continuous cohomology of groups and classifying spaces. *Bull. Amer. Math. Soc.*, 84(4):513–530, 1978.

[Wig73] David Wigner. Algebraic cohomology of topological groups. *Trans. Amer. Math. Soc.*, 178:83–93, 1973.

[Wil70] Albert Wilansky. *Topology for analysis*. Ginn (A Xerox company), Waltham, Mass.-Toronto, Ont.-London, 1970.

[Yon60] Nobuo Yoneda. On Ext and exact sequences. *J. Fac. Sci. Univ. Tokyo Sect. I*, 8:507–576 (1960), 1960.