

# 1 All Topics

1. 4-Dimensional geometry
2. Abacus
3. Algorithms in computer science (e.g. sorting)
4. Mathematics of alternating current
5. Approximating  $\pi$
6. Bernoulli Brothers
7. Bernoulli numbers
8. Billiards
9. Buffon Needle problem
10. The Butterfly Theorem
11. Calculus in economics / biology / chemistry / physics / engineering / some other area
12. Cauchy-Euler Differential Equation
13. Ceva's Theorem and Menelaus's Theorem
14. Charles Proteus Steinmetz
15. Mathematics of Chess
16. Clairaut Differential Equation
17. Collatz conjecture
18. Combinatorics
19. Complex numbers
20. Computer graphics
21. Computing  $\int \sin^n x \cos^m x dx$

22. Compute the digits of  $\pi$  - write your own program!
23. Constructing regular polygons
24. Continued fractions
25. The cross ratio
26. Cryptography/cryptology (secure connections)
27. The Delian problem
28. DeMoivre's formula and/or Chebyshev polynomials
29. Differential equations in chemistry / physics / engineering / some other area
30. Diophantine equations
31. Discuss the history / applications / derivation of  $e$
32. Domino problem: if you remove two opposite corners of a chessboard, can you cover what remains with  $1 \times 2$  dominoes?
33. Dynamical systems and fixed points (possibly even talk about chaotic dynamical systems) only using real numbers
34.  $e$ : is it rational? Is it transcendental?
35. Elementary Number Theory
36. Euclid's *Elements*
37. Euler's formula
38. Euler line
39. Euler-Mascheroni constant
 
$$\gamma = \lim_{n \rightarrow \infty} \left( 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln(n) \right) \approx 0.57721 \dots$$
40. Fermat's little theorem

41. Fermat's work
42. Fermat's last theorem
43. Fibonacci sequence
44. Find a formula for the integral of  $x^n e^{x^m}$  if  $n = km + m - 1$  and  $k$  and  $m$  are positive integers. Prove it by induction.
45. Find some series  $\sum a_n$  such that it is impossible to use any of the tests presented in class to see if these series converge or diverge.
46. Flatland (book)
47. Four-color theorem
48. Fourier series
49. Fractals
50. Game theory
51. Gamma function  $\Gamma$
52. Geometry problem (requires calculus): A circle of radius 1 is drawn; an equilateral triangle is inscribed in it; then a circle is inscribed in the triangle; then a square is inscribed in the circle; then a circle is inscribed in the square; then a regular pentagon is inscribed in the circle, etc. Does the radius of the circles drawn in such way go to 0?
53. Golden ratio/golden spiral
54. Graph theory
55. Graphing conic sections; rotation of axes
56. Group theory
57. Hamilton/Euler circuits, Königsberg bridge problem
58. Homogeneous linear equations with constant coefficients
59. Hyperbolic trigonometric functions

60. “Illegal” summations of series that diverge and techniques of assigning values to some of these, e.g. Cesaro summation
61. Indiana State Legislature’s attempt to make  $\pi$  a rational number
62. Mathematical induction; strong induction
63. Infinite products
64. Infinite sets and cardinalities; Georg Cantor
65. Juggling
66. Klein bottle
67. Knight’s tours
68. Knot theory
69. Latin squares
70. Linear algebra
71. Linear programming
72. Logic & Truth Tables
73. M.C. Escher
74. Magic squares
75. Math & architecture
76. Mathematical modeling using technology
77. Mathematics and medicine
78. Mathematics and music
79. Mathematics of gambling
80. Mathematics of Lewis Carroll
81. Mobius band

82. Modular arithmetic
83. Monte Carlo methods
84. Non-Euclidean geometry
85. NP-complete puzzles/games
86. Number theory
87.  $O$  and  $o$  notation - notation which is used to compare different functions
88. Optical illusions
89. Origami
90. Orthogonal trajectories and differential equations
91.  $\pi$ : is it rational? Transcendental?
92. Pascal's Mystical Hexagram
93. Pappus's Theorem
94. Perfect numbers
95. Planar graphs
96. Planiverse (book)
97. Platonic solids
98. Prime numbers (Mersenne & GIMPS)
99. Prime Number Theorem
100. Projective geometry and conic sections
101. Ptolemy's theorem in geometry
102. Pythagorean theorem: its proofs, uses and history
103. Records in computing  $\pi$
104. Ricatti Differential Equation

105. The Runge-Kutta method
106. Russell's paradox
107. Series solutions to differential equations
108. The Simson Line
109. Simpson's Rule
110. Spherical geometry and the flight of the airplane
111. Statistics
112. Sokoban; PSPACE-completeness vs. NP-completeness
113. Squaring the circle
114. Sudoku
115. Surreal / hyperreal numbers
116. Taxicab geometry
117. Tessellations
118. TI-92 exploration (3-D graphing, geometry software, ...)
119. Topology
120. Tower of Hanoi puzzle
121. Triangle geometry
122. Trisection problem
123. Use of complex numbers and Heaviside's method in solving partial fractions problems
124. Wallis's product formula for  $\pi$
125. Zeno's paradoxes

## 2 Accessible for Students in Precalculus and Lower

1. 4-Dimensional geometry
2. Abacus
3. Algorithms in computer science (e.g. sorting)
4. Mathematics of alternating current
5. Approximating  $\pi$
6. Bernoulli Brothers
7. Bernoulli numbers
8. Billiards
9. The Butterfly Theorem
10. Ceva's Theorem and Menelaus's Theorem
11. Mathematics of Chess
12. Collatz conjecture
13. Combinatorics
14. Complex numbers
15. Computer graphics
16. Compute the digits of  $\pi$  - write your own program!
17. Constructing regular polygons
18. Continued fractions
19. The cross ratio
20. Cryptography/cryptology (secure connections)
21. The Delian problem

22. DeMoivre's formula and/or Chebyshev polynomials
23. Diophantine equations
24. Domino problem: if you remove two opposite corners of a chessboard, can you cover what remains with  $1 \times 2$  dominoes?
25. Elementary Number Theory
26. Euclid's *Elements*
27. Euler's formula
28. Euler line
29. Fermat's little theorem
30. Fermat's work
31. Fermat's last theorem
32. Fibonacci sequence
33. Flatland (book)
34. Four-color theorem
35. Fractals
36. Game theory
37. Golden ratio/golden spiral
38. Graph theory
39. Group theory (may be too difficult!)
40. Hamilton/Euler circuits, Königsberg bridge problem
41. Indiana State Legislature's attempt to make  $\pi$  a rational number
42. Mathematical induction; strong induction
43. Infinite sets and cardinalities; Georg Cantor (may be too difficult!)

44. Juggling
45. Klein bottle
46. Knight's tours
47. Knot theory
48. Latin squares
49. Linear algebra
50. Linear programming
51. Logic & Truth Tables
52. M.C. Escher
53. Magic squares
54. Math & architecture
55. Mathematical modeling using technology
56. Mathematics and medicine
57. Mathematics and music
58. Mathematics of gambling
59. Mathematics of Lewis Carroll
60. Mobius band
61. Modular arithmetic
62. Monte Carlo methods
63. Morley's Theorem
64. Non-Euclidean geometry
65. NP-complete puzzles/games
66. Number theory

67. Optical illusions
68. Origami
69. Pascal's Mystical Hexagram
70. Pappus's Theorem
71. Perfect numbers
72. Planar graphs
73. Planiverse (book)
74. Platonic solids
75. Prime numbers (Mersenne & GIMPS)
76. Projective geometry and conic sections
77. Pythagorean theorem: its proofs, uses and history
78. Records in computing  $\pi$
79. Russell's paradox
80. The Simson Line
81. Statistics
82. Sokoban; PSPACE-completeness vs. NP-completeness
83. Spherical geometry and the flight of the airplane
84. Squaring the circle
85. Sudoku
86. Taxicab geometry
87. Tessellations
88. Topology
89. Tower of Hanoi puzzle

90. Triangle geometry
91. Trisection problem
92. Zeno's paradoxes

### 3 Calculus-Related Topics

1. Alternating current
2. Alternating sums of divergent series
3. Approximating  $\pi$ : in ancient times and now
4. Calculus in economics / biology / chemistry / physics / engineering / some other area
5. Cauchy-Euler Differential Equation
6. Clairaut Differential Equation
7. Computing  $\int \sin^n x \cos^m x dx$
8. Continued fractions
9. Compute the digits of  $\pi$  - write your own program!
10. DeMoivre's formula and Chebyshev polynomials
11. Differential equations in chemistry / physics / engineering / some other area
12. Discuss the history / applications / derivation of  $e$
13. Dynamical systems and fixed points (possibly even talk about chaotic dynamical systems) only using real numbers
14.  $e$ : is it rational? Is it transcendental?
15. Explain the different ways of defining  $e$  and why they are equivalent
16. Exact differential equations

17. Euler-Mascheroni constant

$$\gamma = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln(n)\right) \approx 0.57721 \dots$$

18. Fibonacci sequence

19. Find a formula for the integral of  $x^n e^{x^m}$  if  $n = km + m - 1$  and  $k$  and  $m$  are positive integers Prove it by induction

20. Find some series  $\sum a_n$  such that it is impossible to use any of the tests presented in class to see if these series converge or diverge

21. Fourier series

22. Gamma function  $\Gamma$

23. Golden ratio/golden spiral

24. Homogeneous linear equations with constant coefficients

25. Hyperbolic trigonometric functions

26. Infinite products

27. "Illegal" summations of series that diverge and techniques of assigning values to some of these, e.g. Cesaro summation

28. Indiana State Legislature's attempt to make  $\pi$  a rational number

29.  $O$  and  $o$  notation - notation which is used to compare different functions

30. Orthogonal trajectories and differential equations

31.  $\pi$ : is it rational? Is it transcendental?

32. Records in computing  $\pi$

33. Ricatti Differential Equation

34. The Runge-Kutta method

35. Series solutions to differential equations

36. Simpson's Rule
37. Solve the following problem. A circle of radius 1 is drawn; an equilateral triangle is inscribed in it; then a circle is inscribed in the triangle; then a square is inscribed in the circle; then a circle is inscribed in the square; then a regular pentagon is inscribed in the circle, etc. Does the radius of the circles drawn in such way go to 0?
38. Use of complex numbers and Heaviside's method in solving partial fractions problems
39. Wallis's product formula for  $\pi$

## **4 Original Research Topics for the Advanced Math Topics Class at North Central High School, Indianapolis, IN**

I am indebted to Mr. Paul Brown for this list of topics.

1. 4-dimensional geometry
2. Bernoulli Brothers
3. Buffon Needle problem
4. Charles Proteus Steinmetz
5. Mathematics of Alternating Current.
6. Complex Numbers
7. Computer Graphics
8. Continuous Fractions
9. Creating an NC Calculus Web Page
10. Cryptography/Cryptology-(Secure Connections)
11. Diophantine Equations-Fermat's Thm

12. Fibonacci Series
13. Flatland (book)
14. Four-color theorem
15. Fractals
16. Geometry on the TI-92
17. Georg Cantor
18. Golden Mean/Ratio/Section
19. Graph Theory
20. Hamilton/Euler Circuits (Quaternions)
21. Indiana State Legislature's attempt to make  $\pi$  a rational number
22. Induction Types (e.g. strong induction)
23. Infinite Sets
24. Knot Theory
25. Königsberg Bridge Problem
26. Linear Programming
27. Logic & Truth Tables
28. M.C. Escher
29. Magic squares
30. Math & Architecture
31. Mathematical Modeling using technology
32. Mathematics of Economics
33. Mathematics and medicine
34. Mathematics and music

35. Mathematics of Gambling
36. Mathematics of Lewis Carroll
37. Modular Arithmetic
38. Monte Carlo methods
39. Number Theory
40. Perfect Numbers
41. Planiverse (book)
42. Prime Numbers (Mersenne & Gimps)
43. Ray Tracing
44. Russell's Paradox
45. Statistics
46. Surreal Numbers
47. TI-92 Exploration (3-D graphing, geometry software, ...)
48. Topology
49. Tower of Hanoi Puzzle
50. Trisection Problem
51. Zeno's Paradox