1 General Mistakes

1. $\sqrt{1+x^2}$ is NOT $1+x$! In general, for any two numbers or expressions x and y, $\sqrt{x+y}$ is NOT $\sqrt{x}+\sqrt{y}$!

2. $\frac{1}{1+x^2}$ is NOT $\frac{1}{1}+\frac{1}{x^2}$!!! You can't break up the denominator! You can only break up the numerator.

3. $\cos^{-1}(x) = \arccos(x)$, not $1/\cos(x)$! $1/\cos(x)$ is called $\sec x$! Similarly, $\sin^{-1}(x)$ is NOT $1/\sin(x)$ - that's $\csc x$. And $\tan^{-1}(x)$ is NOT $1/\tan(x)$ - that's $\cot x$. $\cos^{-1}(x)$ is the angle θ such that $\cos(\theta) = x$.

4. $\cos(x+y)$ is NOT $\cos x + \cos y$! $\sin(x+y)$ is NOT $\sin x + \sin y$! Similarly for all trigonometric functions AND their inverses, and for logarithms and exponentials - basically ALL functions! In fact, if f is a continuous function and $f(x+y) = f(x) + f(y)$ for all x and y then $f(x) = ax$, where $a = f(1)$.

5. $\cos(xy)$ is NOT $\cos x \cos y$! Similarly for basically ALL functions! In fact, if f is a continuous function defined for $x > 0$ and $f(xy) = f(x)f(y)$ for all x and y, then $f(x) = 0$ or $f(x) = x^n$ for some number n.

6. Be careful to find the derivative when you need to do that, and find the integral when you need to do THAT!! This is obviously very important, but students get these two confused all the time!!!

7. Read ALL directions on tests carefully!! If it says to find the sum, don't just prove the series converges. If it says to use a specific technique, make sure you use it! Don't forget to do all the parts of the problem!

8. Simplify as much as you can, find all derivatives, and evaluate all trigonometric functions and their inverses if you can.

2 Integrals

1. Watch out: is it a definite or indefinite integral? If there are limits, make sure you plug them in and get a number as the answer! Similarly, if you are asked to find the **particular** solution to the differential

equation, make sure you get a '$+C$' when you integrate and solve for that C!

2. **Integration by substitution**: don't forget to change the limits! Or, do NOT plug them in until you're done integrating AND convert back to the original variable.

3. If you're integrating with respect to u (you have du in your integral), then the answer should come out in terms of u, not x!

4. The antiderivative of a product is NOT the product of the antiderivatives!! In other words, $\int f(x)g(x)\,dx$ is NOT $(\int f(x)\,dx)(\int g(x)\,dx)$!! This is a serious misunderstanding of the product rule for derivatives and shows you don't understand anything about **integration by parts**!

5. Partial fractions: make sure you do long division first if the degree of the numerator is not less than the degree of the denominator. Also make sure you factor the denominator before you start doing partial fractions!

3 Series

1. **Geometric series**: be careful in finding the first term and the ratio. The ratio may be negative - watch out for that! To find the first term, the easiest way may be to plug in the first value of n, for example in $\sum_{n=1}^{\infty} \frac{2^n}{5 \cdot 3^{n-1}}$, plug in $n = 1$ to find that the first term is $2/5$.

2. **Telescoping series**: $\sum_{n=1}^{\infty} f(n) - f(n+1)$ is NOT just $f(1)$! The infinitely many terms after $f(1)$ do NOT just cancel out! You have to find the expression for the N-th partial sum and then take the limit as N goes to infinity.

3. If $\lim_{n\to\infty} a_n = 0$, then it tells you NOTHING about $\sum a_n$. This means you need to try another approach! It's only when $\lim_{n\to infty} a_n$ is NOT 0 that the series $\sum a_n$ diverges. And it does diverge if the limit $\lim_{n\to\infty} a_n$ does not exist.

4. You can't split up $\sum(a_n \pm b_n)$ into $\sum a_n \pm \sum b_n$ unless both $\sum a_n$ and $\sum b_n$ converge! Specifically, you can't tell that $\sum(a_n + b_n)$ diverges

just because $\sum a_n$ diverges or $\sum b_n$ diverges. For example, $\sum(1)$ and $\sum(-1)$ both diverge, but $\sum(1+(-1)) = \sum(0) = 0$, hence converges. For a less trivial example, $\sum_{n=1}^{\infty}(\frac{1}{n} - \frac{1}{n+1})$ converges to 1 (it's a telescoping series - work it out!) but $\sum \frac{1}{n}$ and $\sum \frac{1}{n+1}$ both diverge - they're the harmonic series (up to a term). It is true, however, that if $\sum a_n$ diverges and $\sum b_n$ converges, then $\sum(a_n \pm b_n)$ diverges.

5. Any time you find a limit of a sequence, ask yourself what you were finding the limit of. WHAT DOES THE NUMBER YOU GET REALLY TELL YOU??? Was it the sequence of terms? Then you're trying to use the Divergence / Limit / n-th Term Test or the Alternating Series Test. Was it the sequence of partial sums? Then you're actually computing the sum of the series. Was it $\lim_{n\to\infty} \frac{a_n}{b_n}$? Then you're using the limit comparison test to see if the two series $\sum a_n$ and $\sum b_n$ behave similarly. Was it $\lim_{n\to\infty} \sqrt[n]{|a_n|}$ or $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}|$? Then you're using the root or ratio test.

6. **(Direct) Comparison Test**: If you find your series $\sum a_n$ is less than $\sum b_n$ but $\sum b_n$ diverges, then there is no conclusion. Similarly, if you find your series $\sum a_n$ is larger than $\sum b_n$ but $\sum b_n$ converges, then there is no conclusion.