CS310: ALGORITHMS AND DATA STRUCTURES

Theta Notation: Formal Definition

- When we say $T(n)$ is $\Theta(f(n))$, we mean that $T(n)$ is sandwiched between two constant multiples of $f(n)$ i.e. $c_1 f(n) \leq T(n) \leq c_2 f(n)$ for $c_1, c_2 > 0$
- This claim applies when n is sufficiently large, i.e. some $n \geq n_0$

Theta Notation *Cont* …

- Even though we write $T(n) = \Theta(f(n))$, as in $T(n) = \Theta(n^2)$, $\Theta(f(n))$ is actually a set of functions
- $\Theta(f(n))$ represents a set of all functions having the same rate of growth as $f(n)$, so
- $\Theta(f(n)) = \{T(n)$: there exist positive constants c_1, c_2 such that $c_1 f(n) \leq T(n) \leq c_2 f(n)$ for all $n \geq n_0$ where n_0 is some threshold value$\}$
- $\Theta(n^2) = \{n^2, n^2-1, \frac{1}{5}n^2, \frac{1}{5}n^2+3n, an^2+n\log n \ldots\}$

Theta Notation *Cont* …

- Thus saying that $T(n) = \Theta(n^2)$ really means that the function $T(n) = ½n^2 - 3n$ € $\Theta(n^2)$ where

$\Theta(n^2)$ = set of all those f(n) such that there exist positive constants c_1, c_2 for which $c_1 f(n) \leq T(n) \leq c_2 f(n)$ for all $n \geq n_0$ where n_0 is some threshold value

Asymptotic Notations

- We have already seen Theta (Θ) notation
- There are other asymptotic notations
  - O: the big-oh
  - Ω: the big-omega
  - o: the little-oh
  - ω: the little-omega

- n >= 0, T(n) >= 0, n (domain) is continuous

Big Oh: Formal Definition

- When we say T(n) is O(f(n)), it means that T(n) is always less than or bounded from above by some multiple of f(n) i.e. $T(n) <= c_1 f(n)$ for $c_1 > 0$
- This claim applies when n is sufficiently large, i.e. some $n >= n_0$
- Note: if $T(n) = \Theta(f(n))$ then $T(n) = O(f(n))$
- But : $T(n) = O(f(n))$ =/=> $T(n) = \Theta(f(n))$, why?

Example

- $an^2+bn+c = O(n^2)$, then there should exist a constant $c_2>0$, such that for large n, $an^2+bn+c <= c_2n^2$

- O(f(n) is the set of all functions with the same or smaller order of growth than f(n)
- $O(n^2) = \{n^2, 100n + 5, \log n, \ldots\}$
- Why n^3 or n^4 do not belong to $O(n^2)$?

Big Omega: Formal Definition

- When we say T(n) is Ω(f(n)), it means that T(n) is bounded from below by some multiple of f(n) i.e. $c_1 f(n) <= T(n)$ for $c_1 > 0$
- This claim applies when n is sufficiently large, i.e. some $n >= n_0$
- T(n) = Θ(f(n)) => T(n) = Ω(f(n))

  but not the converse

Example

- $an^2+bn+c = \Omega(n^2)$, then there should exist a constant $c_2>0$, such that for large n, $an^2+bn+c >= c_2n^2$
- $c_2 = a/2$

- $\Omega(n^2)$ is the set of all functions with a larger or the same order of growth as f(n)
- Why $6n+100$ is not $\Omega(n^2)$?