Counting Twin Primes in Residue Classes

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Primes

Residue classes for n mod 4

- $4x + 0 = 4x = 0, 4, 8, 12, 16, \ldots \equiv 0 \bmod 4$
- $4x + 1 = 1, \underline{5}, 9, \underline{13}, \underline{17}, 21, 25, \underline{29}, 33, \underline{37}, \underline{41}, 45, 49, \ldots \equiv 1 \bmod 4$
- $4x + 2 = 2(2x + 1) = \underline{2}, 6, 10, 14, 18, \ldots \equiv 2 \bmod 4$
- $4x + 3 = \underline{3}, \underline{7}, \underline{11}, 15, \underline{19}, \underline{23}, \underline{27}, \underline{31}, 35, 39, \underline{43}, \underline{47}, \ldots \equiv 3 \bmod 4$

Prime races

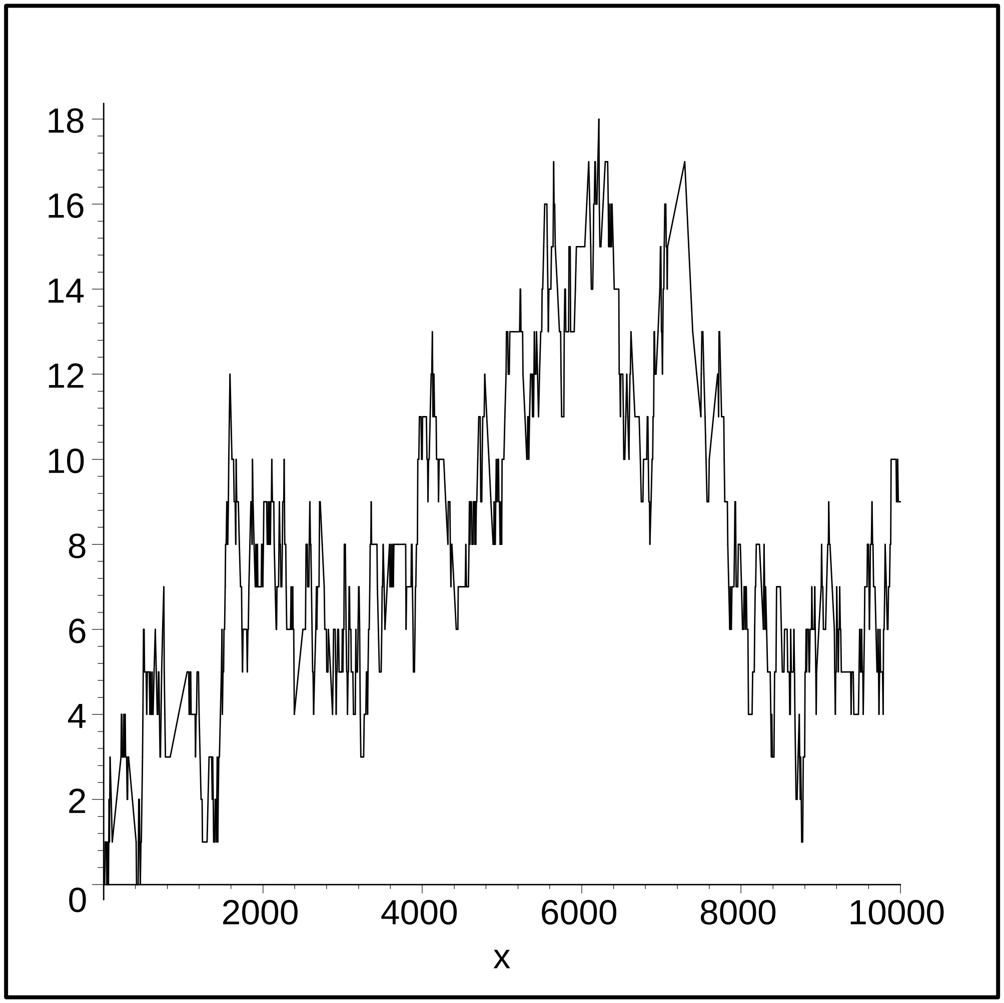


Figure 1: The difference between the number of primes which are 3 mod 4 and 1 mod 4 up to x

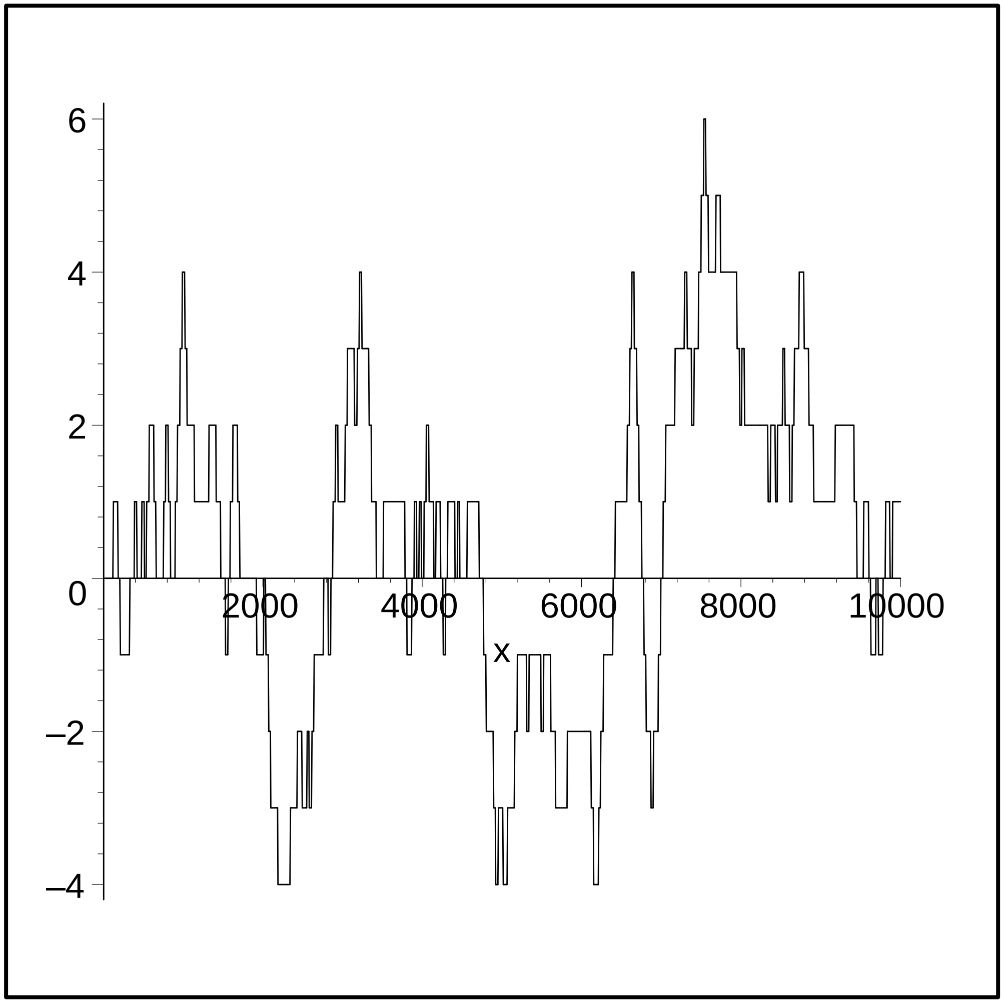


Figure 2: The difference between the number of primes which are 7 mod 30 and 13 mod 30 up to x

Primes

- $4x + 1 = 1, \underline{5}, 9, \underline{13}, \underline{17}, \ldots$
- $4x + 3 = \underline{3}, \underline{7}, \underline{11}, 15, \underline{19}, \ldots$

Weighted Count

- $\frac{\pi_{4,1>3}(20)}{\ln 20} = \frac{1}{3} + \frac{1}{4} \approx \frac{0.58333}{ln20} \approx 0.19472$
- $\frac{\pi_{4,3>1}(20)}{ln20} = \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \frac{1}{15} + \frac{1}{16} + \frac{1}{19} + \frac{1}{20} \approx \frac{1.03336}{ln20} \approx 0.34494$
- $\frac{\pi_{4,1=3}(20)}{\ln 20} = \frac{1}{2} + \frac{1}{5} + \frac{1}{6} + \frac{1}{17} + \frac{1}{18} \approx \frac{0.98104}{\ln 20} \approx 0.32748$

$$\sum_{i=p_k}^{p_{k+1}} \frac{1}{i} \approx \ln(p_{k+1}) - ln(p_k) + \frac{1}{2}(\frac{1}{p_k} - \frac{1}{p_{k+1}})$$

Twin Primes: p, $p+2$ prime.

for example, $p = 3$, $p+2 = 5$

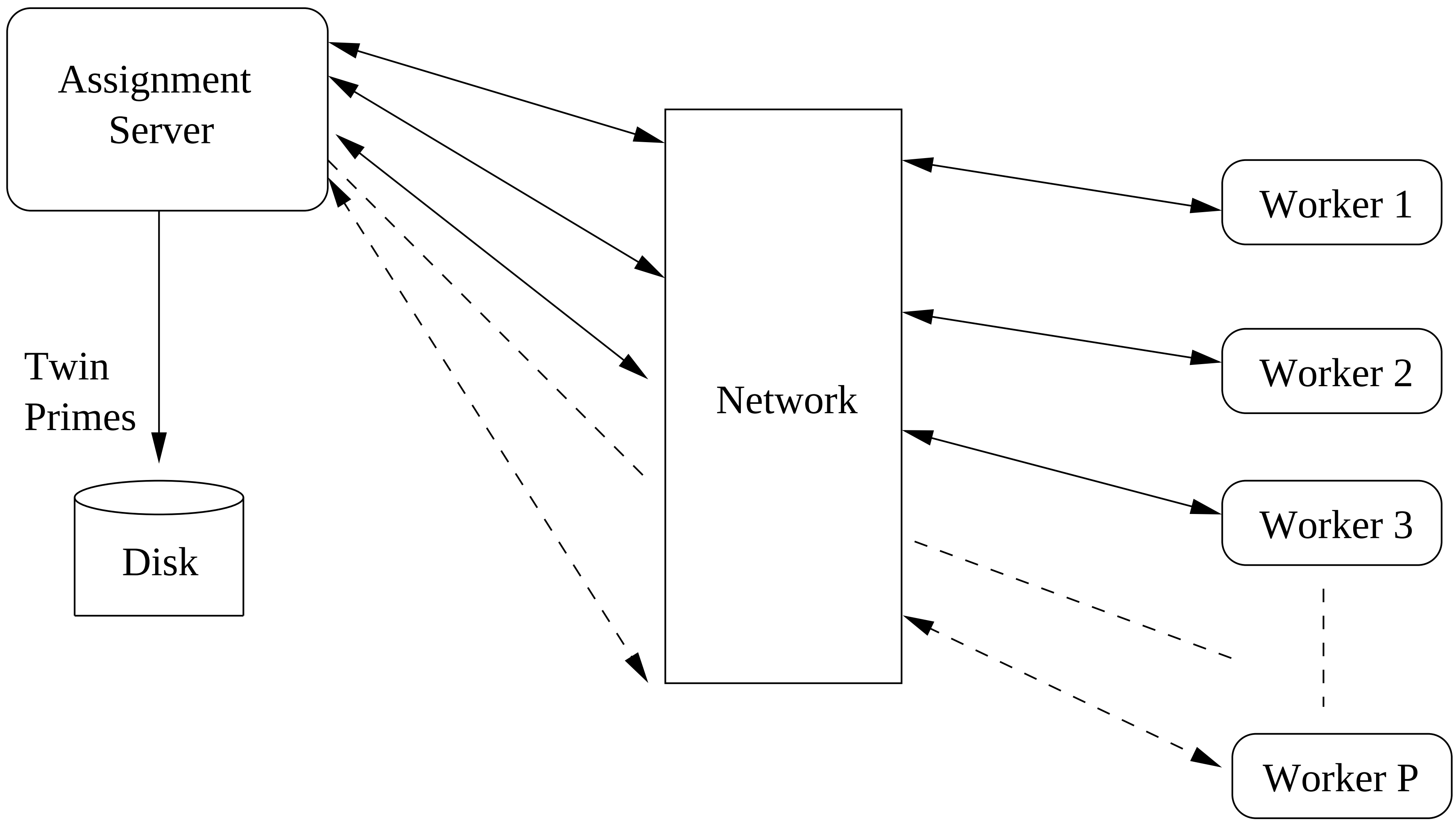


Figure 3: Interactions between the assignment server and workers

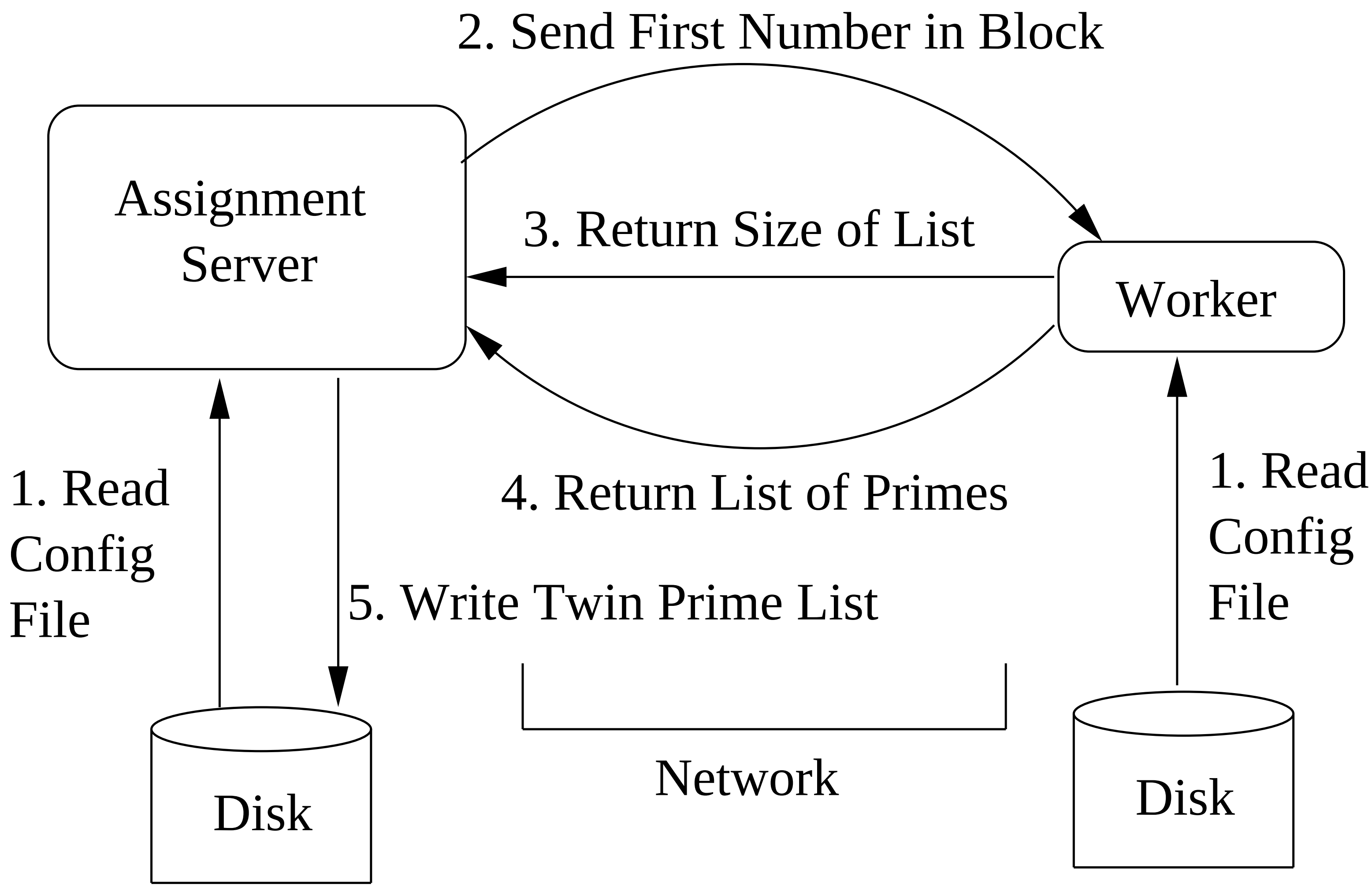


Figure 4: Interactions between the assignment server and one worker. The steps are labeled in order. Steps 2 through 5 repeat until there are no blocks remaining

Time

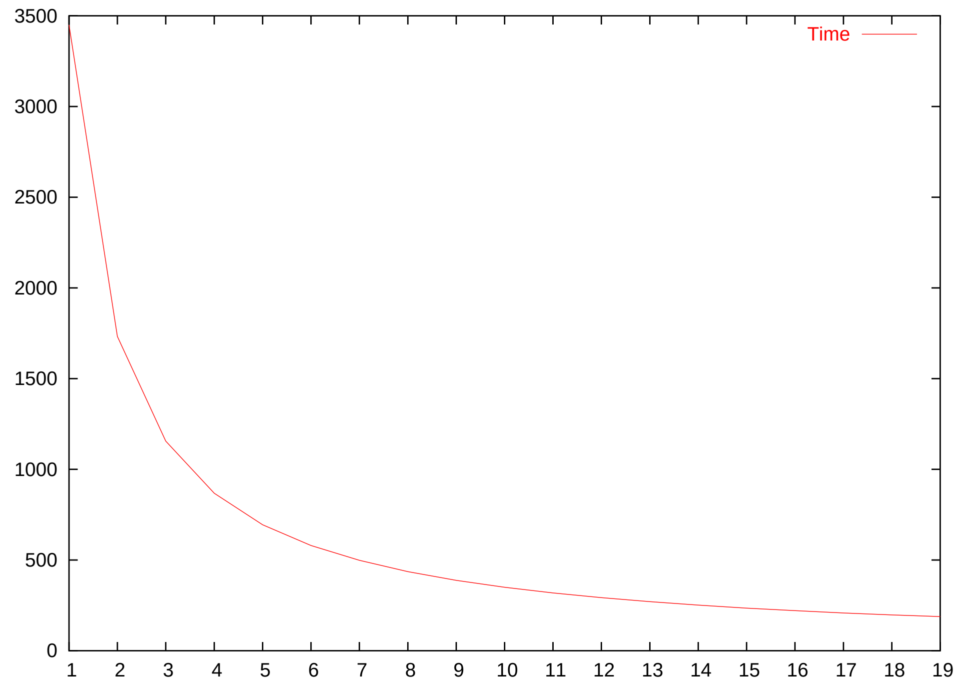


Figure 5: Time in seconds to calculate twin primes out to 10^{11} for 1 to 19 worker processors

Speedup

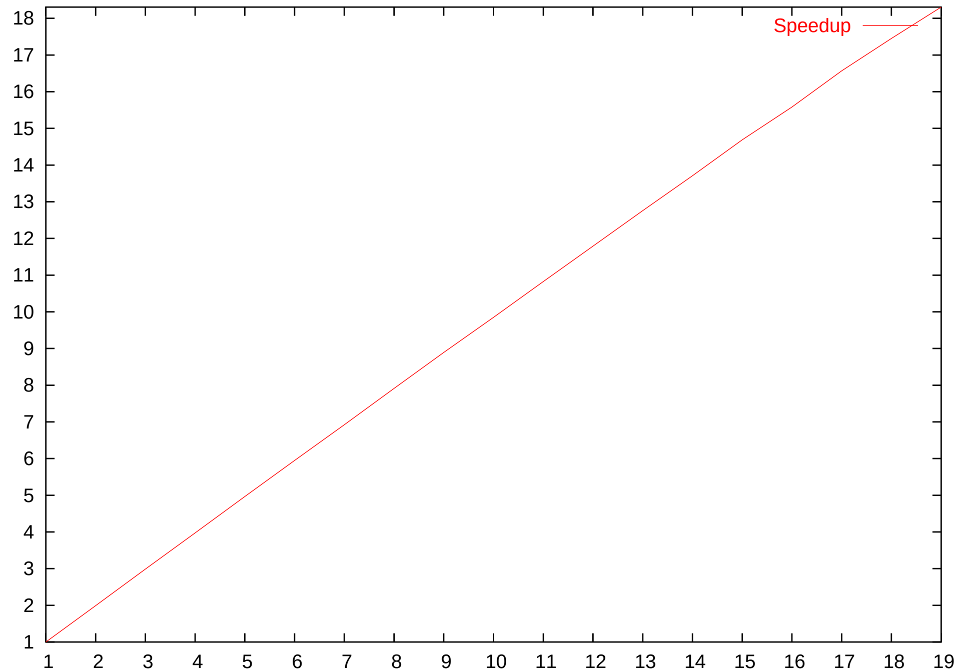


Figure 6: Speedup for 1 to 19 worker processors

Speedup(p) =amount of time for one processor/amount of time for p processors

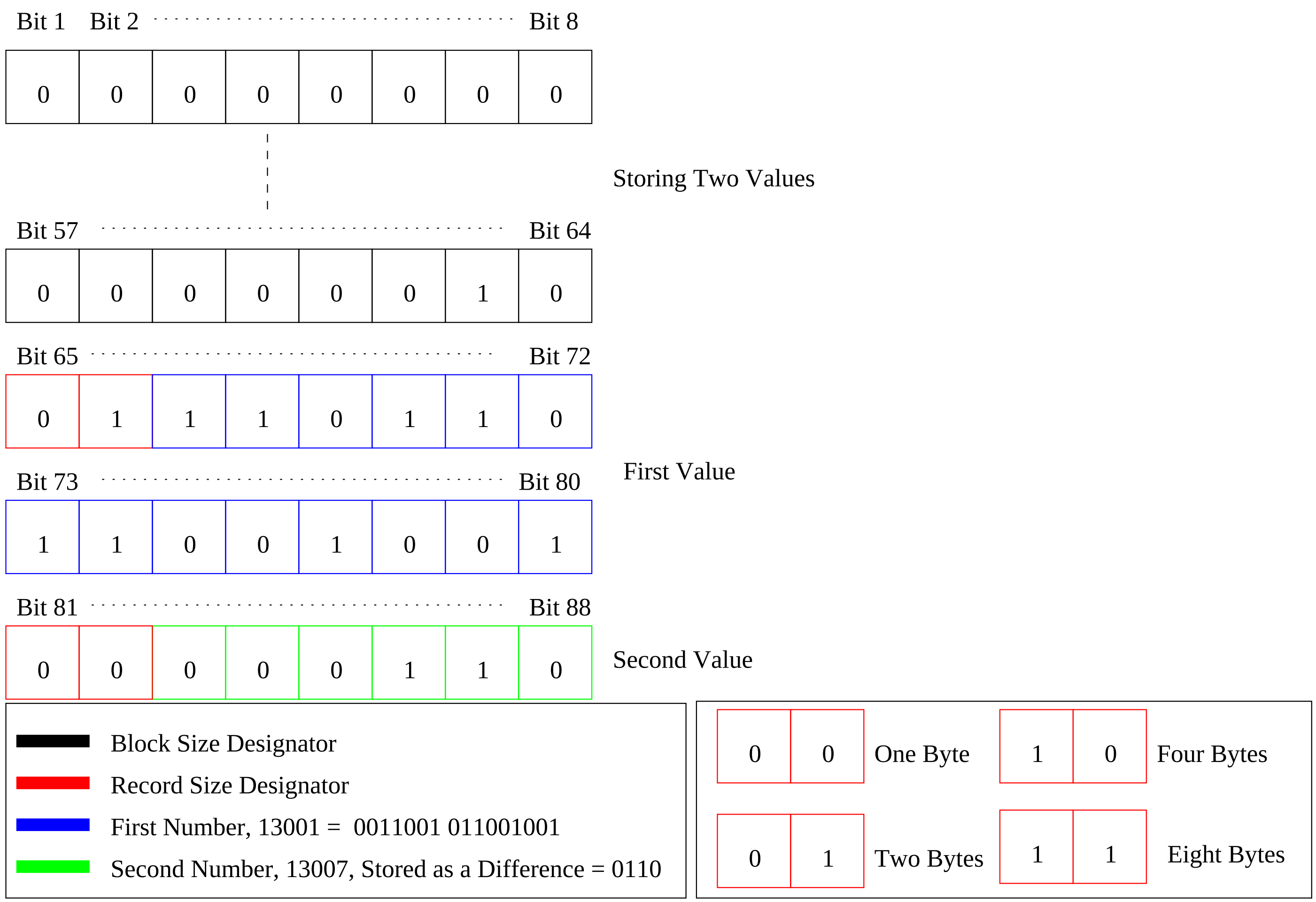
$$\pi_2(n) \leq c\Pi_2 \frac{n}{(\ln n)^2}(1 + O\frac{\ln\ln n}{\ln n}) \approx c\Pi_2 \frac{n}{(\ln n)^2}$$

$$\text{storage}(n) = 64\pi_2(n)$$

$$\text{storage}'(n) = \sum_{k=1}^{n}(\ln(p_k - p_{k-1})) \approx$$

nth twin prime $\approx n\ln(n)^2$

$$\sum_{k=1}^{n}(\ln(k(\ln k)^2 - (k-1)(\ln(k-1))^2) \approx 2n\ln\ln n$$


Bit 1 Bit 2 Bit 8
0 0 0 0 0 0 0 0
Storing Two Values
Bit 57 Bit 64
0 0 0 0 0 0 1 0
Bit 65 Bit 72
0 1 1 1 0 1 1 0
First Value
Bit 73 Bit 80
1 1 0 0 1 0 0 1
Bit 81 Bit 88
0 0 0 0 0 1 1 0
Second Value
Block Size Designator
Record Size Designator
First Number, 13001 = 0011001 011001001
Second Number, 13007, Stored as a Difference = 0110
0 0 One Byte
1 0 Four Bytes
0 1 Two Bytes
1 1 Eight Bytes

Results

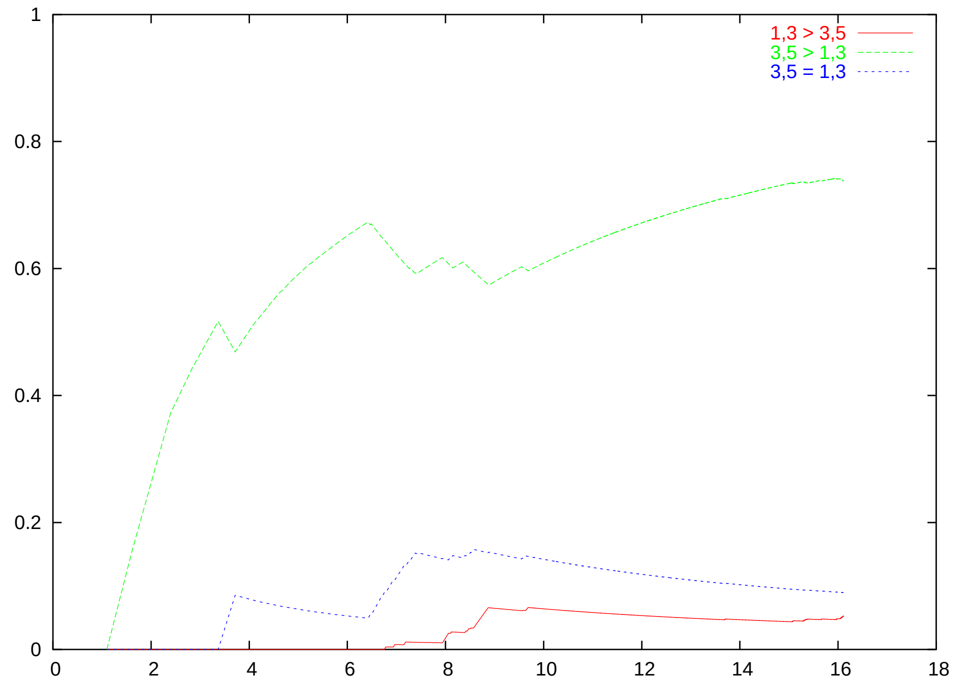


Figure 7: $1, 3 \mod 8$ vs $3, 5 \mod 8$

Results

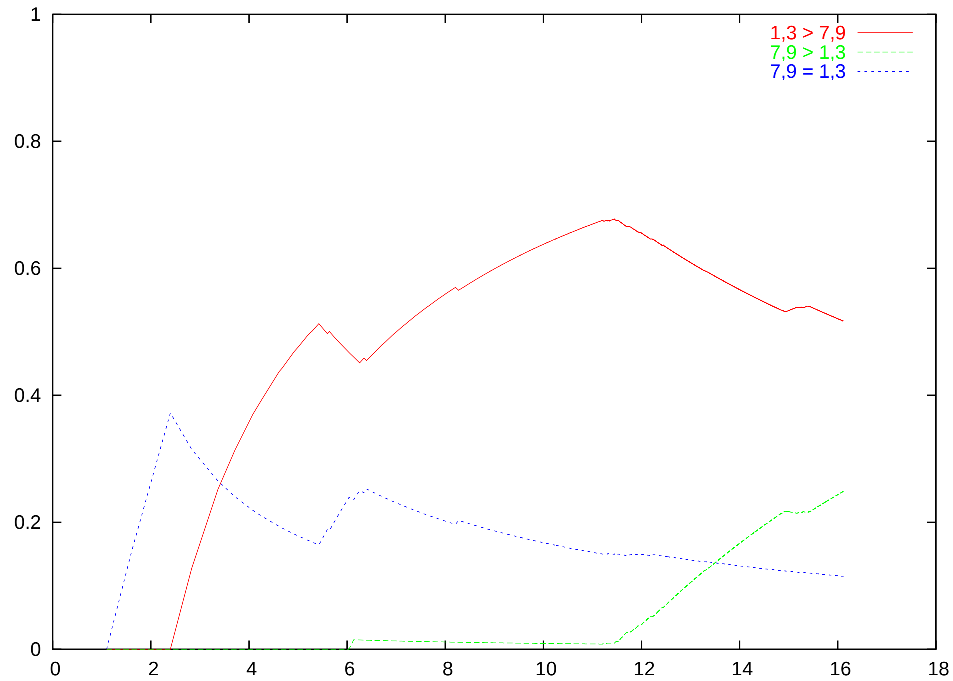


Figure 8: $1,3 \mod 8$ vs $7,1 \mod 8$