Population Models

CCG, Earlham College

Thomas Malthus' Population Model

- $P(t) = P(t-1) + rP(t-1)$
- $P(t)$: Population at time t
- r: rate of population growth
- $r = r_b - r_d$
- r_b: birth rate, r_d death rate
- i.e., $r_b = .5, r_d = .1 \implies r = .4$

Solving at any time.

1. For $t > 0$:
   $P(t) = P(t-1) + rP(t-1)$
   $= P(t-1)(1+r)$

2. For $t = 0$:
   $P(0) = P_0$, constant representing the population when $t = 0$.

3. $P(1) = P_0(1+r)$

4. $P(2) = P(1)(1+r) = (P_0(1+r))(1+r) = P_0(1+r)^2$

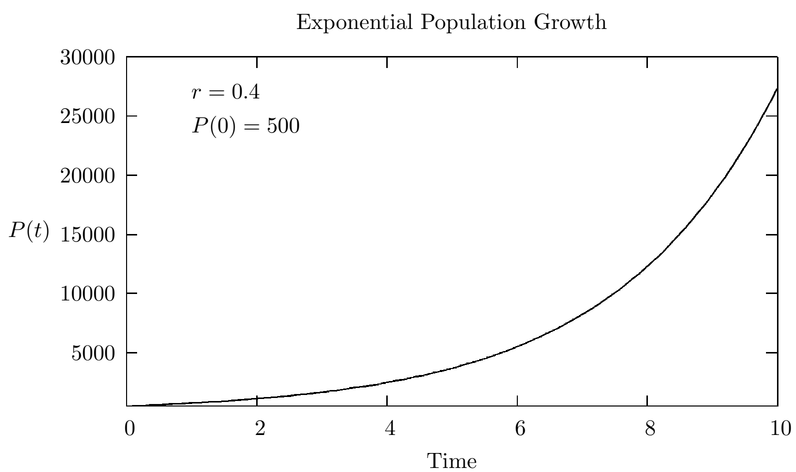
5. $P(3) = P(2)(1+r) = (P_0(1+r)^2)(1+r) = P_0(1+r)^3$

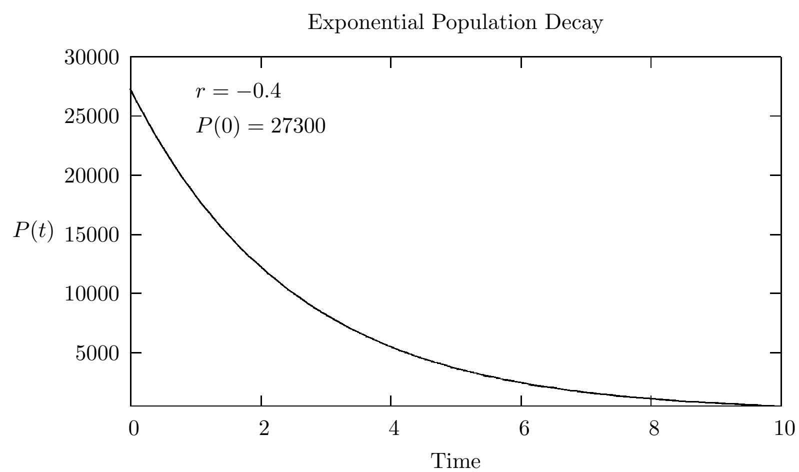
6. Generalizing…

7. $P(t) = P_0(1+r)^t$

Exponential Growth Model

"There is no exception to the rule that every organic being naturally increases at so high a rate that, if not destroyed, the earth would soon be covered by the progeny of a single pair."
–Charles Darwin





Sheep code

```
ask sheep [
           move
           reproduce-sheep
           death
]
```

Doubling Time

1. $P(t) = P_0(1+r)^t$
2. $2P(0) = P_0(1+r)^t$
3. $P_2 = t = \frac{\ln 2}{\ln(1+r)}$
4. For our example, $r = .4, P_0 = 500$. $P_2 = 2.06$

Validation & Verification of the Malthusian Growth Model

What's missing from this model?

Verhulst Growth Model

1. Previous Expontential Model: $P_0(1+r)^t$

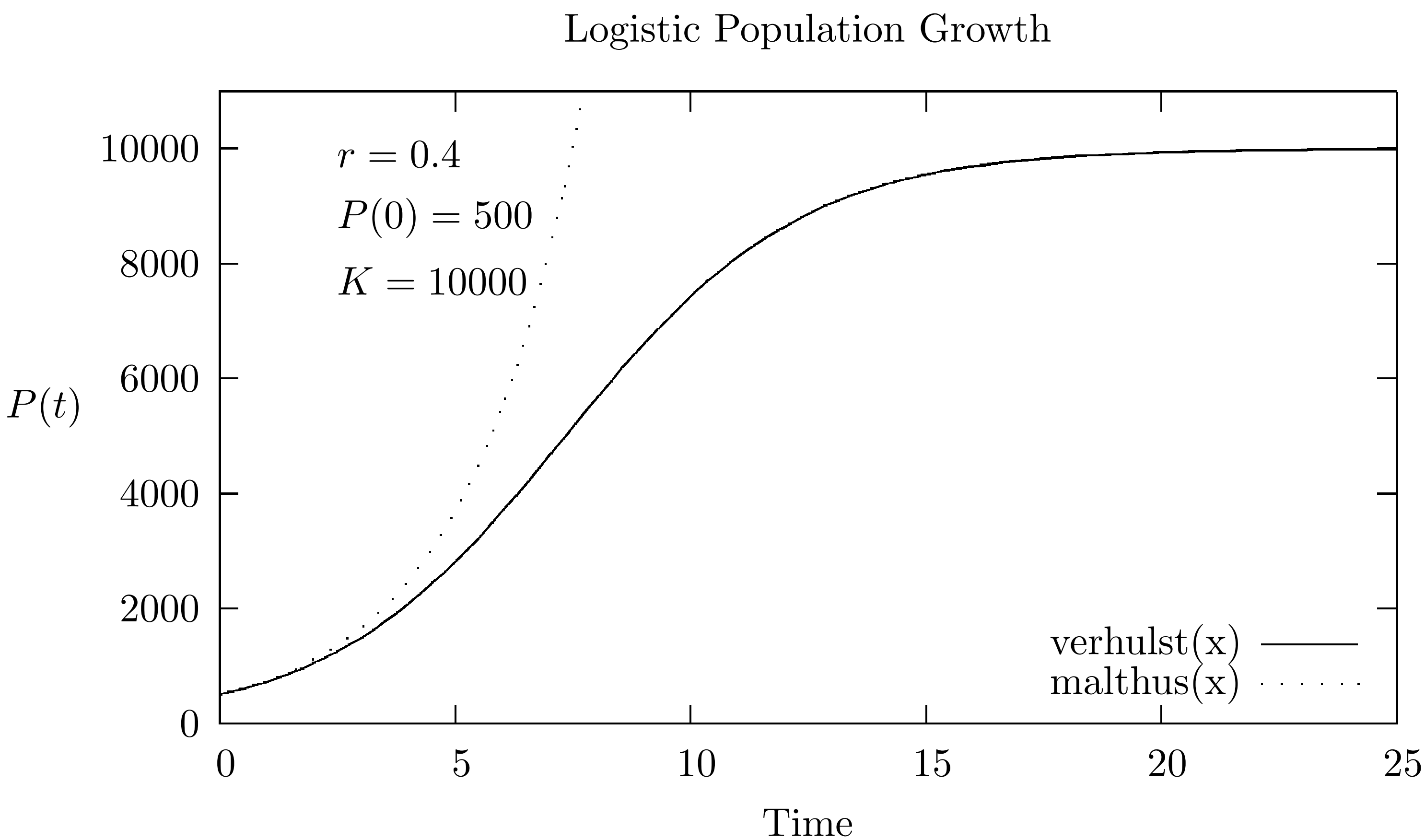
2. Continuous Model: $P(t) = \frac{KP_0(1+r)^t}{K+P_0(1+r)^t-1}$

3. K: Carrying capacity of environment.

4. r_0: Growth rate for small populations.

5. $$\frac{KP_0(1+r)^t}{K+P_0(1+r)^t-1}\frac{\frac{1}{P_0(1+r)^t}}{\frac{1}{P_0(1+r)^t}} \rightarrow \frac{K}{\cancel{\frac{K}{P_0(1+r)^t}}+\cancelto{1}{\frac{P_0(1+r)^t}{P_0(1+r)^t}}-\cancel{\frac{1}{P_0(1+r)^t}}} \rightarrow K$$

Verhulst Growth Model



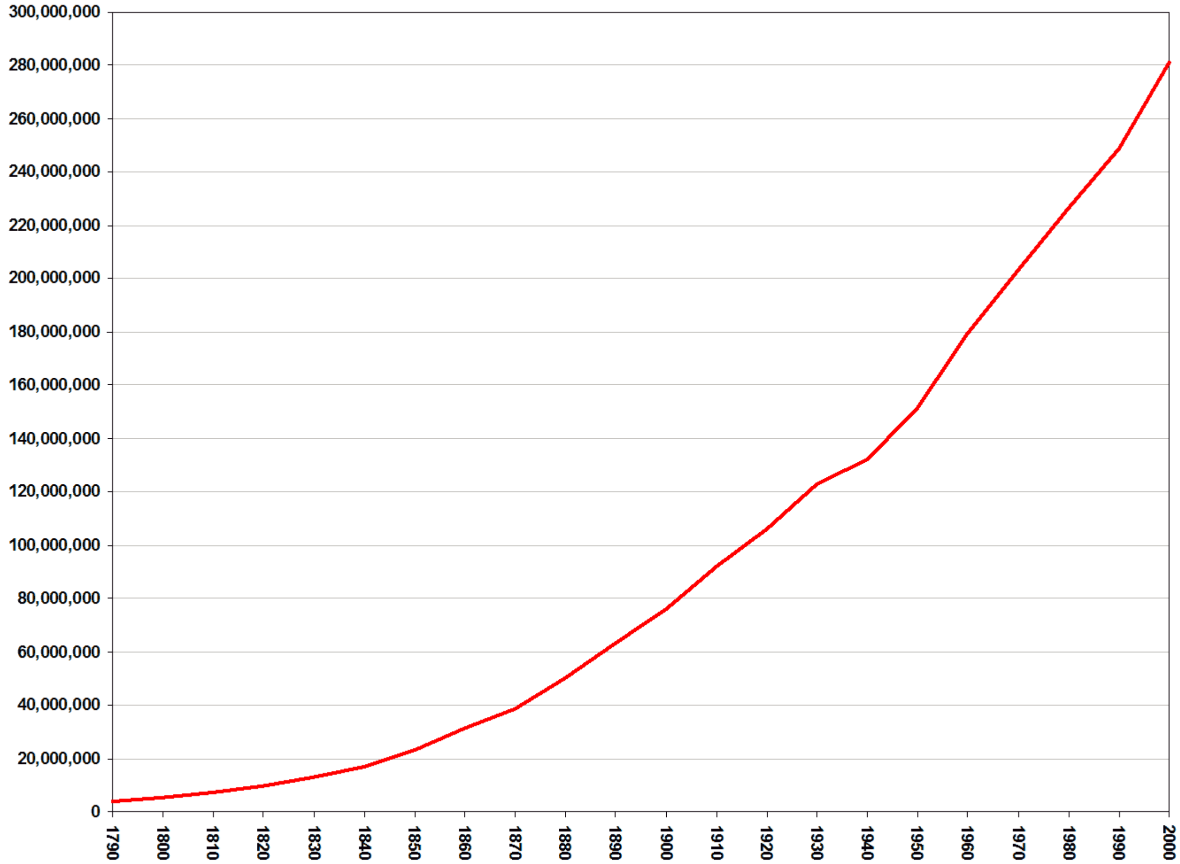
Sheep code 2.0

```
ask sheep [
           move
           if grass? [
             ;; deduct energy for sheep only if
             ;;    grass? switch is on
             set energy energy - 1
             eat-grass
           ]
           reproduce-sheep
           death
]
```

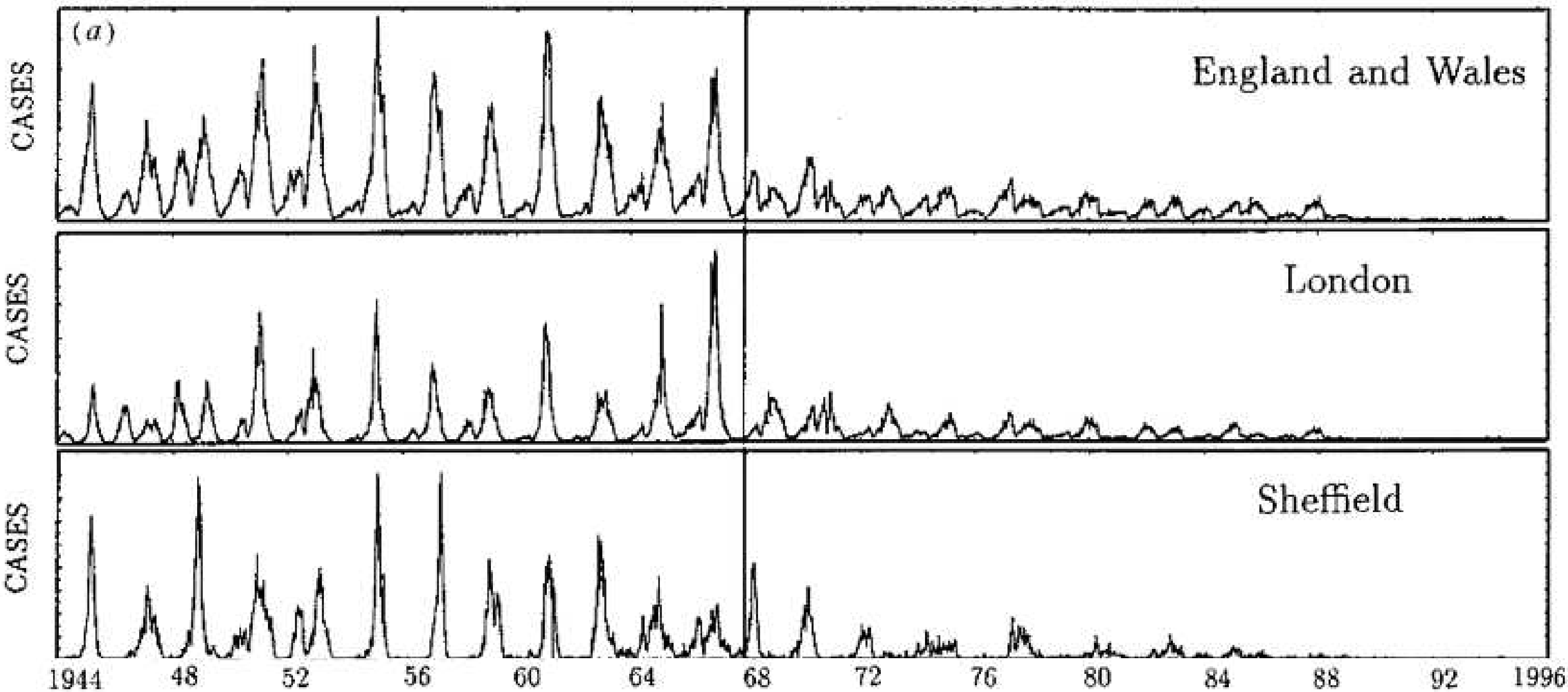
Validation & Verification of the Verhulst Growth Model

What's missing from this model?

US Population from 1790 to 2000



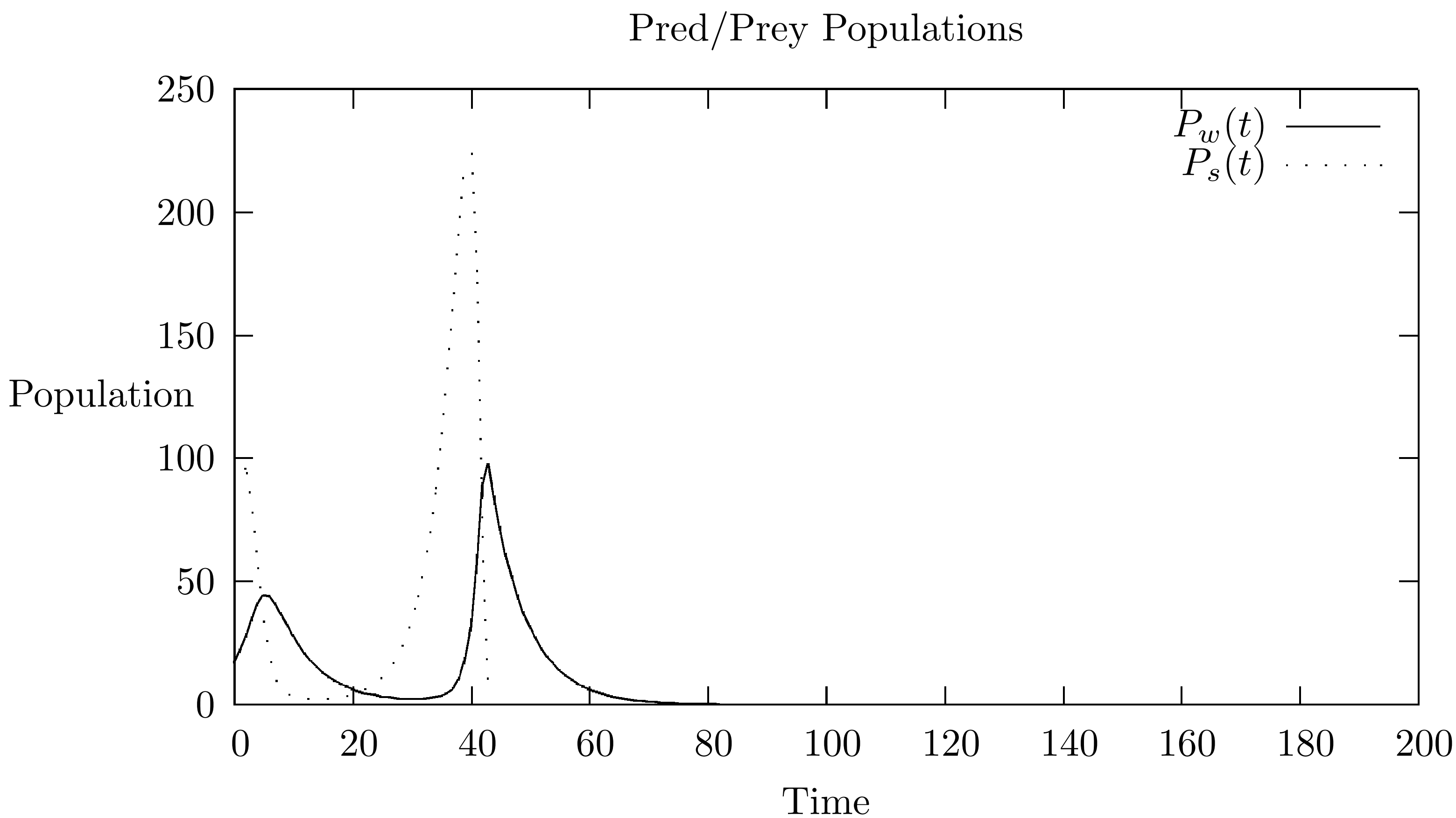
Predator/Prey Models: Measles In England 1944-1996



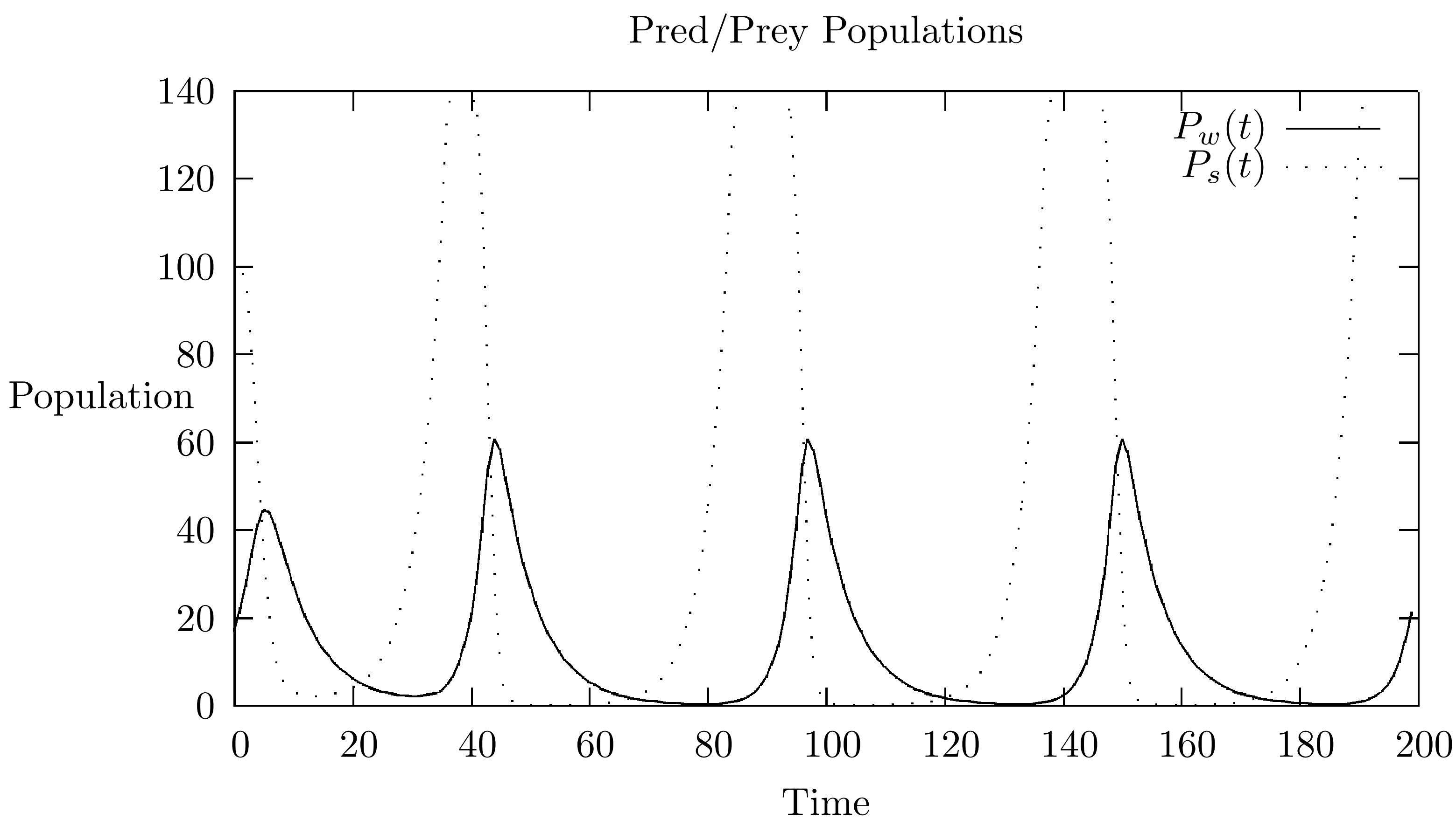
Lotka-Volterra Difference Equations: Sheep & Wolves

1. Original Model: $P(t) = P(t-1) + rP(t-1)$
2. $P_s(t) = P_s(t-1) + r_s P_s(t-1) - kP_s(t-1)P_w(t-1)$
3. $P_w(t) = P_w(t-1) + r_w P_w(t-1) + ekP_s(t-1)P_w(t-1)$
4. r_s: birth rate of sheep with no predators (> 0).
5. r_w: death rate of wolves with no prey. (< 0).
6. $P_s(t-1)P_w(t-1)$, encounters between wolves and sheep.
7. k: the rate at which wolves kill sheep they encounter.
8. e: efficiency of turning eaten sheep into a new wolf.

Iterative Results



Iterative Results with maximum sheep population of 140.



Wolf code

```
ask wolves [
  move
  set energy energy - 1  ;; wolves lose energy as they move
  catch-sheep
  reproduce-wolves
  death
]
```

V & V of the Lotka-Volterra Model

What's missing from this model?